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A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

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APPLIED MECHANICS REVIEWS

VOL. 4, NO. 10 MARTIN GOLAND *Editor* OCTOBER 1951

DEVELOPMENT OF THE MATHEMATICAL THEORY OF THREE-DIMENSIONAL PHOTOELASTICITY

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THE FIRST NINETY YEARS

IN THE first systematic treatment of the laws relating strain and optical fields, Neumann (1)*, in 1841, gave not only the complete linear relation between the small strain and index tensors, but also approximate ordinary differential equations governing the rectilinear propagation of a ray of light in the anisotropic and heterogeneous optical medium resulting from the application of heterogeneous strain. Although this was a great stride toward the establishment of a theory on which an experimental method of three-dimensional stress analysis could be based, many questions were left unanswered; especially, what optical measurements should be made and how they should be interpreted.

Neumann's extraordinary accomplishment remained unused and unimproved for many years, despite a pregnant hint from Maxwell's (2) experiment with isinglass in 1850 which failed to stimulate an early development of the freezing method. The first reappearance of Neumann's results is found in the great treatise of Coker and Filon (3) ninety years later. Here Neumann's strain-optical relations are extended to finite strain and his differential equations are rederived and utilized in a discussion of optical effects accompanying the quadratic variation of stress through the thickness of a plate in a state of plane stress.

Occupying a unique position between the old and new regimes in photoelasticity is an interesting paper by Poritzky (4) in 1934. Although dealing with a two-dimensional problem of thermal stresses in long coaxial cylinders, Poritzky gave a correct interpretation of the integrated birefringence along a chord an essentially three-dimensional optical technique which has been the subject of several recent investigations.

RESURGENCE

The current activity in three-dimensional photoelasticity was stimulated during the period 1936–1940 by reports and interpretations of experiments with the freezing and slicing technique by Oppel (5), Hetényi (6), and Kuske (7); studies of stress-optical relations and measurements with convergent light by Hiltscher (8); and the introduction of the scattered-light technique by Weller (10) and Menges (11). From the point of view of the mathematical theory, Hiltscher's paper is important for containing the discovery that the two-dimensional state of stress can be resolved completely by optical measurements involving relative, rather than absolute, phase retardation, while the paper by Menges reports the first use of Neumann's differential equations in the analysis of experimental data.

LAWS OF PROPAGATION OF LIGHT

Whereas Neumann's differential equations are based on purely kinematical considerations, involving approximations of hidden character and magnitude, the electromagnetic theory combined with Neumann's strain-optical or Maxwell's stress-optical relations supplies the general field equations governing the propagation of light in the anisotropic, heterogeneous medium encountered in three-dimensional photoelasticity. The latter equations, being a system of partial differential equations with variable coefficients, are formidable in appearance; but they have been solved for two cases of special interest in the interpretation of photoelastic observations, namely, when the magnitudes only and the orientations only, of the principal axes of the sections of the index ellipsoid, normal to the ray, vary linearly along the path of the ray. The first of these exact solutions, by Mindlin (13), led to the conclusion that variation in magnitude alone has no unusual influence for index variations at rates ordinarily encountered. The second solution, by Drucker and Mindlin (12), revealed some curious and useful effects of rotation of polarizing axes on the light vector and also led to the establishment of a "rotation-retardation ratio" as a criterion for assessing the influence of rotation on the relation between relative phase retardation and secondary principal stress difference. They found this influence to be small.

When terms of negligible magnitude are dropped from the two exact solutions, they may be shown to be identical with the corresponding solutions of Neumann's equations. Thus, approximations to exact solutions of the general electromagnetic equations are identical with exact solutions of Neumann's approximate equations in the two cases for which direct comparisons can be made. That this will always be the case was shown by

* Numbers in parentheses refer to the Bibliography at the end of the paper.

EDITOR'S NOTE: The tenth in a series of articles, contributed to APPLIED MECHANICS REVIEWS by international authorities, surveying important topics in applied mechanics.

Mindlin and Goodman (27) who executed the passage between the equations themselves, showing, in the process, the nature and order of magnitude of the approximations required, and establishing the generality of the rotation-retardation ratio employed by Drucker and Mindlin. Similar results were obtained recentiv by O'Rourke (34) who applied the Sommerfeld-Runge method for deriving laws of geometrical optics from the electromagnetic equations.

RESOLUTION OF PRINCIPAL STRESSES

Measurements involving relative rather than absolute phase retardations at interior points were shown by Hiltscher (8) to yield the three principal stress differences in the general case and the principal stresses themselves in two-dimensional states of stress. He applied point by point measurements with convergent polarized light to both cases (8, 19), as also did Kuske (16). Drucker and Mindlin (12) extended this idea to observations on whole sections of the model by suggesting passage of light in an appropriate number of directions either in the plane of the section or at an angle to it. These preliminary studies culminated in the development by Drucker (18, 31) of the "method of oblique incidence."

Many of the methods for resolving principal stresses in two dimensions have counterparts in three dimensions. Those already studied are measurements of change of thickness of a reheated frozen slice by Kuske (7); graphical integration of the stress equations of equilibrium along stress trajectories (in a plane of symmetry) by Jessop (24); and in rectangular coordinates by Frocht and Guernsey (35). Neuber's method could also be extended to three dimensions, and the various methods of integragtin Laplace's equation could be employed to find the sum of the three principal stresses.

FLEXURE OF PLATES

In the problem of flexure of plates, the essential difficulty is that of the optical antisymmetry with respect to the middle plane, resulting in cancellation of relative phase retardation. A number of sandwich and layer devices have been employed to eliminate the antisymmetry and reduce the optical situation to an essentially two-dimensional one. However, the simplest solution has been in the direction of making the system more three-dimensional. This reversal of trend resulted from a suggestion by Mindlin (13) to freeze an initial tension in the plate so that the subsequent application of bending would produce an asymmetrical rather than an antisymmetrical optical situation. Mindlin proposed to measure the resulting birefringence by using the phenomenon of rotation of the light vector previously discovered by Drucker and Mindlin (12), but Drucker (17) found that greater precision and simplicity would be gained by observing the fringe pattern in the ordinary transmission polariscope. Drucker's measurements of stress concentration around a hole and subsequent comparison (20) of his results with the predictions of E. Reissner's new theory of flexure constitute both an interesting sequence of events and a landmark in the accomplishments of three-dimensional photoelasticity.

TORSION

One of the simplest three-dimensional states of stress occurs in a cylindrical bar under torsion. This case is an excellent one for the beginner to study in order to become acquainted with some of the complexities of three-dimensional photoelasticity. The optical properties of a bar in torsion are described by Mindlin (9), and the identity of scattered-light fringe patterns with the contours of the membrane analogy is demonstrated by Drucker and Frocht (22) and Saleme (23)

INTEGRATED PHASE RETARDATION

A very intriguing idea in three-dimensional photoelasticity is the possibility of measuring only the integrated relative phase retardation, after complete passage through the model or specimen and finding the stresses at interior points by using auxiliary mathematical theory. This would dispense with the tedious freezing-slicing procedure and the difficult scattered-light technique. Considerable progress has been made along these lines since the early work on cylindrical bars by Poritzky (4). An ingenious study by Read (28) of stresses in glass bulbs accomplishes this result in a special case of much interest in industry. Indications that the complete problem has some possibility of solution are contained in a remarkable series of papers by O'Rourke and Saenz (32, 33, 34). Considering only the cases of light passed through equatorial planes of symmetrically strained cylinders and spheres, they show that the gross retardation can be expressed in terms of the interior stresses by means of an Abel integral equation Since this equation can be solved, they find simple quadrature formulas for the interior stresses.

PHOTOVISCOELASTICITY

Most experiments in photoelasticity are actually performed with viscoelastic materials, but little work has been done, and that only recently, in the formulation and solution of mathematical equations relating viscoelastic and optical properties. In a paper by Mindlin (25) are given general relations connecting stress, strain, time, temperature, and birefringence. These are specialized to incompressible media and are used in finding conditions under which the birefringence in viscoelastic materials may be interpreted in terms of the stress in an elastic body. It is found that this interpretation is valid except if there are moving external loads. Another conclusion reached is that the freezing method in three-dimensional photoelasticity is not restricted to models made of cross-linked polymers. Although this may appear surprising, it should not be, in view of Maxwell's (2) one-hundred-year-old experiment with isinglass. Mindlin's results were subsequently revised and extended by Read (30) who included compressible media, inertia terms, and any type of boundary condition. Both papers are limited to small strain and linear viscoelasticity.

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Communications

Concerning AMR 4, Rev. 1172: Luen B. Banks, Machine for testing rails in bending fatigue.

The theory given in this paper is not in error if interpreted correctly. Masses can be replaced by equivalent forces if the fact that the dependent variable appears with the forcing terms is borne in mind, and appropriate care taken. The frequency equation deduced is in error and applies only to a beam without attached masses. If required, the equation for the natural frequency of the beam with attached mass can be deduced if the elimination of the dependent variable from the forcing terms is effected before a zero denominator is sought. The error mentioned does not affect the particular application as the natural frequency is introduced only to obtain a convenient dimensionless parameter for the purpose of calculation.

Luen B. Banks, England

Theoretical and Experimental Methods

(See also Revs. 3840, 3939, 3964, 3967)

3746. Rinebolt, J. A., and Harris, W. J., Jr., Statistical analysis of tests of Charpy V-notch and keyhole bars, *Weld. Res. Suppl.* **16,** 4, 202s–208s, Apr. 1951.

From fifteen closely equivalent heats of a laboratory steel (composition 0.30 C, 1.0 Mn, 0.30 Si) 300 Charpy V-notch and 300 keyhole impact bars were tested over a range of temperatures. Transition temperatures (according to various definitions) were determined from plots of energy and fibrous area vs. temperature. Statistical analysis of these data enabled the calculation of the standard deviation of each transition temperature as a function of the definition and the number of available bars. For the standard deviation of the "average energy" transition temperature (determined with 20 V-notch bars, spaced over the entire transition range) a value of 6.7 F was found. This means a necessary difference between steels for 95% statistical significance of 19.0 F. For the determination of the "15 ft-lb level" transition temperature, with the same standard deviation, only 8 V-notch bars, tested over a limited temperature range, are necessary. An appendix, dealing with the influence of C, Si, and Ni on the transition temperatures, leads to the conclusion that standard deviations are not greatly changed by changes in the composition of carefully made laboratory heats.

J. H. van der Veen, Holland

3747. Nemes, T., Mechanical solution of Diophantic problems, *Müeg. Közl.* no. 1, 29–50, 1949.

Paper contains a proposal for the construction of an automatic machine for the solution of the two-move chess problem. Schematic descriptions of the electronic components of such a machine are given.

H. Polachek, USA

3748. Soroka, Walter W., Resistance network analogue for solving vibration problems, *Prod. Engng.* **22,** 4, 103–105, Apr. 1951.

Author presents well-known resistance network analogs for

solving certain algebraic equations. Equations are those determining parameters of steady-state solutions resulting from application of a sinusoidal forcing term to constant-coefficient linear differential equations arising in vibration problems. Basic schemes for multiplication and addition are discussed, followed by treatment of dynamic vibration absorber as an example. Techniques for handling simultaneous systems and increasing accuracy are discussed. Reviewer believes that, where electronic differential analyzer is available, direct solution of linear differential equation is more satisfactory. L. L. Rauch, USA

3749. Qvarnström, B., An analogue computer for systems of linear differential equations, with special reference to problems of aircraft stability, *Instrum. Measur. Confer., Stockholm, Trans.*, 245–249, 1949.

Paper describes design of flexible computer for Royal Institute of Technology, Stockholm. Integrating and summing components are d-c feedback amplifiers with gain of about 5000 in the frequency band 10 to 10,000 cps, d-c gain of 1500, and zero drift of less than 10 millivolt/hr. Square wave inputs are used to provide repeated solutions, as in Philbrick [*Soc. auto. Engrs. quart. Trans.* **2**, 2] and MacNee [*Proc. Inst. Radio Engrs.* **37**, 11, 1315–1324, Nov. 1949] computers. No indication is provided of method used to establish zero level between solutions. Oscillograph is used as output. Paper contains short discussion of stability characteristics of computer. E. Bromberg, USA

3750. Brownell, F. H., III, Nonlinear delay differential equations, *Contr. theory Nonlinear Oscill.*, Study 20, 89–148, 1950.

Paper is concerned principally with proving the existence of periodic solutions of some types of nonlinear ordinary difference-differential equations. In order to do so, a review and, in part, a new presentation of facts about linear systems of this type are carried out. In addition, certain methods devised by E. Schmidt to deal with nonlinear integral equations are extended so that they can be applied to the present problem. The results are too complicated to be formulated in a review of reasonable length.
J. J. Stoker, USA

3751. Diliberto, S. P., On systems of ordinary differential equations, *Contr. theory Nonlinear Oscill.*, Study 20, 1–38, 1950.

New results concerning the following three problems are established: I. The reduction, by linear transformations, of systems of first-order linear differential equations with variable coefficients to diagonal or triangular form. II. Geometric criteria for stability or periodic solutions (closed trajectories) of systems of first-order nonlinear differential equations. III. Bounds on the number of periodic solutions of a system of first-order differential equations with polynomial functions.
From author's summary by J. J. Stoker

3752. van Heemert, A., On the numerical evaluation of certain types of integrals, *Nat. LucătLab. Amsterdam Rap.* F.55, 21 pp., 1949.

Integrals arise in calculation of downwash of airfoils. They are: $Q_n = {}_0\int^\pi (\cos n\theta)/(\tau)\, d\theta$; $U = {}_0\int^\pi (\theta \sin \theta)/(\tau)\, d\theta$; $T_n = {}_0\int^\pi (\cos n\theta)/(\tau^3)\, d\theta$; $\lambda_n = {}_0\int^\pi (\sin n\theta \sin \theta)/(\tau^3)\, d\theta$; $J_0' = {}_0\int^\pi (\theta \sin \theta)/(\tau^3)\, d\theta$; $Q_n^{\xi_0} = {}_0\int^\pi (\cos n\theta)/(\xi - \xi_0)\tau\, d\theta$; $U^{\xi_0} = {}_0\int^\pi (\theta \sin \theta)/(\xi - \xi_0)\tau\, d\theta$; where ${}_0\int^\pi$ denotes the principal value of the integral in the Cauchy sense, $\tau^2 = (\xi_0 - \xi)^2 + \eta_0^2$; $1 - \xi = \cos\theta$, and ξ_0 and η_0 are given.

Each integral is evaluated by several methods, one method being selected as suitable for numerical evaluation. For example: Q_n is found (1) by expansion in terms of elliptic integrals, (2) by reduction formula on changing to prolate elliptic coordinates, (3) in terms of a series involving Legendre polynomials. The last is most suitable for calculation. Other integrals are evaluated in similar ways or in terms of those already found. Numerical calculations to follow.
C. C. Gotlieb, Canada

3753. Sneddon, I. N., Fourier transforms, New York, Toronto, London, McGraw-Hill Book Co., Inc., 1951, xii + 542 pp. $10.

Book has been written "for those whose interest is primarily in the applications of the theory rather than in the theory itself." From this point of view, author has, in general, done a competent job. Mathematical results, together with assumptions which are made to prove these results, are clearly stated. This latter factor is often conspicuously missing in many books on applied mathematics.

The first three chapters are devoted to the theory of various types of integral transforms. The standard theorems, including convolution theorems, are proved for Fourier, Laplace, and Mellin transforms under the assumptions that the functions involved belong to L_1 ($\int |f|\, dx < \infty$) and satisfy the Dirichlet condition (i.e., they have only a finite number of maxima and minima, a finite number of finite discontinuities, and no infinite discontinuities in their domains). The inversion theorem for Hankel transforms is proved for the class of functions which belong to L_1 and are of bounded variation.

The Dirac delta function is introduced in the first chapter. Even though emphasis is put upon the fact that the manipulations with this function are entirely formal, the language in places is so vague that even one who does not demand too much rigor might feel hesitant about its use. Author seems to be unaware that this important concept has been given a sound mathematical basis by L. Schwartz ["Théorie des distributions," Hermann & Cie, Paris, **I**, 1950; **II**, 1951]. The third chapter concerns the problem of finite transforms.

The remaining seven chapters concern applications of integral transformations to physical problems. A listing of chapter headings gives some indication of the scope of these chapters: The theory of vibrations; The conduction of heat in solids; The slowing down of neutrons in matter; Hydrodynamic problems; Applications to atomic and nuclear physics; Two-dimensional stress systems; and Axially symmetrical stress distributions. These chapters contain not only classical material, but also many recent results, including many of the author's own researches. Three appendixes are attached which contain properties of Bessel functions, approximate methods of calculating integral transforms, and tables of integral transforms.

Reviewer has noted several lapses of the usual clear style in the last seven chapters. The main difficulties arise through the use of "infinitesimals," which often leads to vague statements and, in many instances, gives the impression that less is being assumed than is actually needed. However, this does not detract too much from the general excellence of the book. Reviewer believes that researchers interested in obtaining a working knowledge of integral transforms, and graduate students who are studying applied mathematics should find this work a most valuable contribution. A. Devinatz, USA

3754. Higgins, T. J., An epitomization of the basic theory of the generalized Schwartz-Christoffel transformations as used in applied physics, *J. appl. Phys.* **22**, 3, 365–366, Mar. 1951.

Author presents a tabulation of known results concerning the Schwartz-Christoffel transformation and a bibliography with twenty-six entries. Eugene F. Trombley, USA

3755. Vyalov, S. S., and Kogan, L. G., Solution of some heat problems by methods of electric analogies (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk*, no. 1, 39–52, Jan. 1951.

The electrical analog of resistance networks is applied to two-dimensional steady-state and transient heat transfer in solids. Particular reference is made to the correlation between network arrangement and boundary configuration and to the design of the electrical system simulating steady and time-dependent boundary conditions. Analysis and data are primarily of an illustrative nature. Newman A. Hall, USA

3756. Middleton, D., Noise and nonlinear communication problems, *Symp. appl. Autocorr. Anal. Phys. Probl.*, 24–44, 1949.

Paper is a useful summary of the application of the correlation theory to noise in electric communication network, treated by the author in detail in previous papers. First part outlines the correlation-function analysis. Second part deals with the principal physical problems of noise in electric networks which is changed by different processes (e.g., linear and quadratic rectifying, filtering, cutting of amplitudes). In the last part, exact or approximate solutions, respectively, are given for eight different examples of the most important cases of noise in communication systems. Margot Herbeck, Germany

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 3810, 3825, 3826)

3757. Neményi, P. F., Recent developments in inverse and semi-inverse methods in the mechanics of continua, Advances appl. Mech. **II,** 123–151, 1951. [New York, Academic Press, Inc., $6.50.]

Author calls inverse "an investigation of a partial differential equation of physics if in it the boundary conditions (or certain other supplementary conditions) are not prescribed at the outset. Instead, the solution is defined by the differential equation and certain additional analytical, geometrical, kinematical, or physical properties of the field. In the semi-inverse method some of the boundary conditions are prescribed at the outset, whereas others are left open and obtained indirectly." This article does not attempt to present the subject exhaustively, but aims rather "to elucidate the nature, value, and the potentialities of this approach." Incidentally, however, it explains the essence of nearly all the main contributions.

The paper is divided into sections of about equal length concerning inviscid incompressible fluids, viscous incompressible fluids, perfect gases, elastic bodies in equilibrium, and plastic bodies. Much of the work mentioned is very recent, some unpublished. The author in many cases is able to correlate, compare, and contrast the various investigations, often in different fields. At the end of the paper is a table summarizing results known up to the present on five semi-inverse problems in seven domains of continuum mechanics. Among the more interesting investigations cited are those of Taylor and Trkal on decaying motion of a viscous fluid; those of Tollmien, Prim, and Neményi-Prim on limiting lines, problems of invariance, and "generalized Beltrami flows" in gas dynamics; and those of Neményi on an "influence principle" and on stress trajectories in elasticity.

Author emphasizes the value of simple exact solutions as typical and suggestive illustrative cases, and he shows that such solutions have been or can be obtained by inverse or semi-inverse methods in many cases. His examples, too numerous to list here, substantiate also the following conclusions. "1. Inverse and semi-inverse methods [may] lead to solutions of important boundary-value problems." "2. [They] may lead to the discovery of unsuspected discontinuities, limitations, or general field properties. . ." "3. [They] may settle existence questions in a positive sense, or may decide a uniqueness problem in a negative sense." "4. [They] are essential for the comparative study of the differential equations of the various problems of mechanics."

It is unfortunate that so valuable and stimulating a work has been so carelessly printed. There is an abundance of misprints and grammatical errors, and for the running heads the Academic Press has invented the new term "continua mechanics."

C. Truesdell, USA

3758. Corben, H. C., and Stehle, P., Classical mechanics, New York, John Wiley & Sons, Inc., 1950, xiii + 388 pp. $6.50.

An excellent treatise for the mathematical physicist and others in related subjects. New approach is used to expose underlying assumptions and aid continuous transition from classical to present-day mechanics. Useful exercises are given at the end of each chapter; unfortunately, solutions are not given.

Readers are shown, progressively, theory of vectors, covariants and contravariants, linear operators, matrixes, quaternions, tensors, contact transformations, Poisson brackets, Lagrange brackets, integral invariants, and bilinear covariants. Mathematics is developed and applied to wide range of problems in particle, rigid, and generalized dynamics; electrical and magnetic field theory; orbital motion; atomic and molecular theories; cylindrical magnetron; Rutherford scattering; quantum mechanics; harmonic oscillators; statistical mechanics; perturbation theory; cosmic radiation; thermodynamics; calorimic and thermometric coefficients; special relativity; Thomas precession; betatron; synchrotron; synchro-cyclotron.

First ten chapters deal with particles, rigid bodies, conservative systems, Lagrange's equations, linear vector spaces, and Hamiltonian theory. Later chapters cover contact transformations, Poisson brackets, infinitesimal contact transformations, further developments of transformation theory, miscellaneous generalizations and analogies, introduction to special relativity theory, and the motion of particles in high energy accelerators. No details of quantum mechanics are discussed; only the formal aspects of relativistic theory are introduced; and linear vector spaces are restricted to real number vector components.

"Little of the content is original although much of the presentation is new." Reviewer considers book contains generalizations of concepts of quasicoordinates, Lagrangians, Hamiltonians, Poisson brackets, and Lagrange brackets.

Authors are to be congratulated on the clarity of their new exposition and the publishers on their presentation.

Sidney Kirkby, England

3759. Oldenburger, R., Mathematical engineering analysis, New York, The Macmillan Company, 1950, ix + 426 pp. $6.

Aim of book is stated as: To assist in expressing physical situations in the form of equivalent mathematical relations. Little attempt is made to explain the mathematical processes involved in solving an analytical statement; all the emphasis is placed on determining a significant mathematical formulation for the physical problem.

Mathematics required to understand the book is not minimized. Vectors, line, and surface integrals, Green's and Stokes' theorems, partial differential equations, advanced calculus in general, are assumed understood. References to this background material follow each chapter.

Five essentially unrelated parts make up the book: Mechanics of rigid bodies, electricity and magnetism, heat, elasticity, and fluid mechanics. Basic physical principles associated with each are developed or described, then used to set up a variety of special problems of engineering interest. Problems are realistic and usually contain typical numerical values. Frequent comments are

made concerning regions of validity of assumptions or accuracies obtainable with the necessary simplifications used.

Following each part is a list of problems with which the reader is to set up the mathematical equivalent to the physical system described, but not to solve the equations so obtained. Typical are these: A railroad car runs against a bumper at the end of a track; a pile of powdered coal is burning; a ship rocks on a calm ocean. Answers and hints are given at the end of the book.

Such a book can be equally useful to the mathematician faced with the problems of physical applications, as was Dr. Oldenburger, or to the engineer attempting to relate his physical intuitions to mathematical techniques. Ability to translate from a practical situation to an analytical statement and conversely is often sadly lacking in our graduate scientists today. This critical deficiency is the heart of the book. Industrial problems presented to a mathematician or engineer are never clearly defined, often are scarcely recognizable as problems.

Use of the book as a college or university text will require an excellent mathematical background on the part of the student and an unusual understanding and sympathy for the author's objectives on the part of the professor. Book is a long step forward and it is hoped that its influence will be felt widely.

E. T. Welmers, USA

3760. Noskov, N. I., Spherical construction of positions of a spatial seven-bar linkage (in Russian), *Akad. Nauk SSSR Trudi Sem. Teorii Mash. Mekh.* **9**, 33, 53–71, 1950.

The mechanism is a seven-link linkage with seven turning pairs. The problem is to determine the positions of all the links if the positions of two of them are given. The method is one of graphical interpolation (projections on two perpendicular planes are used for graphical constructions). Geometric loci are indicated by means of rows of points, and their intersection determined by interpolation. These geometric loci are essentially the sets of positions of an axis A_5 of an open chain $A_1A_2A_3A_4A_5$ when A_1 is fixed, and of the open chain $A_5A_6A_7$ when A_7 is fixed. The method is theoretically trivial and practically worthless because of the admittedly huge volume of drafting involved (50 hours). The analytic method of Dimentberg [see AMR **4**, Rev. 3472], in the reviewer's opinion, supersedes this paper completely.

A. W. Wundheiler, USA

3761. Garnier, R., Lectures in kinematics (Cours de cinématique), I, II, III, Paris, Gauthier-Villars, 1949, 1951; iv + 235 pp.; viii + 287 pp.; xi + 376 pp. Fr. 1750, 2000, 3000.

Volume I (Kinematics of a point and of a solid; composition of movements) is essentially a vector treatment of differential geometry somewhat like textbooks by Blaschke and Weatherburn. Emphasis is placed on kinematic methods of generation of curves and surfaces. Subjects considered include the formulas of Frenet and Binet, curvature, torsion, geodesics, and geodesic curvature. In the compositions of motions are included the composition of screw motions, their velocities and accelerations, and the theorem of Coriolis for the composition of accelerations. The Euler angles in the transformation of axes are derived.

Volume II (Rolling and rotation; the Savary formula and its extension to space) shows that every motion of a plane body over another fixed plane may be described uniquely as the rolling of one curve over another fixed curve. Then the Euler-Savary formula can be used to determine the radius of curvature of a path traced by a point in the moving plane body. The construction of Bobillier is demonstrated for the determination of the centers of curvature. The methods are extended to the rolling of one spherical curve over another.

The motion of a body through three-dimensional space may be represented as the rolling of a unique ruled surface over another unique ruled surface. Various properties of these surfaces are considered, including a further analytical extension of the Euler-Savary formula, curvatures, axes of curvature, geodesic curvatures of special curves on these surfaces, and envelopes of carried planes.

Volume III (Cayleyian geometry and kinematics) is concerned with the extension of the Euclidean geometry ideas of Volume II to the non-Euclidean spaces of contant curvature—both elliptic and hyperbolic spaces. Most of the results are carried over with little or no modification. Although of great mathematical interest, they do not seem to be of particular value to the engineer.

M. Goldberg, USA

3762. Grüss, G., On the kinematics of roll-gliding (in German), *Z. angew. Math. Mech.* **31**, 4/5, 97–103, Apr./May 1951.

It is supposed that a movable plane curve is roll-gliding along a fixed curve in such manner that the two curves always have the same tangent at point of contact. Moreover, let the rate of the two lengths of arcs between the points of contact at the time $t = 0$ and at the time t be equal to λ, independent of the time. Thus, $\lambda = 0$ means an absolute gliding and $\lambda = 1$ an absolute rolling; other values $0 < \lambda < 1$ mean a motion denoted "roll-gliding." To investigate the kinematic relations of this roll-gliding, the sets of the two pole curves—the fixed pole curves as well as the movable pole curves of the moving solid—are evaluated for all states of motion, from absolute gliding to absolute rolling. These kinematic relations are discussed in several particular cases.

W. Kochanowsky, Germany

3763. Chang, C.-H., Generalization of the simple reverted planetary spur gear train (in German), *Sci. Rec. Shanghai* **2**, 4, 417–423, Nov. 1949.

Train considered is the common arrangement consisting of coaxial fixed gear, planet carrier, and output gear plus two planetary gears (compounded), one meshing with the fixed gear and the other with the output gear. Author first eliminates the fixed gear and its meshing planetary gear, substituting a pin-jointed link connecting the fixed link and moving planetary plane. It is shown that this results in a varying angular velocity of the output gear for a constant velocity input. Author's explanation of design for obtaining different output motions (including dwell periods or reversal periods) is most valuable part of paper, in reviewer's opinion.

Other arrangements are described briefly. Greatest "generalization" retains all four gears but separates the axes, connecting the four motion planes by a multiplicity of links giving necessary constraint.

A. S. Hall, USA

3764. Bottema, O., and Beth, H. J. E., Euler's equations for the motion of a rigid body in *n*-dimensional space, *Proc. kon. Ned. Akad. Wet. Ser. A*, **54**, 106–108, 1951.

A certain system of $\frac{1}{2}n(n-1)$ equations is given as the appropriate generalization of Euler's well-known equations for a rigid body in three-dimensional space.

D. C. Lewis, USA

3765. Marsicano, F. R., Relation between the Lagrange parameters and the reactions at the constraint in a rigid body which rolls without slip on a plane (in Spanish), *Cienc. y Tec.* **116**, 586, 150–154, Apr. 1951.

Author determines the values of the Lagrangian undetermined coefficients λ in the nonholonomic case of rolling without slip on a plane and finds them to be identical with the components of the reaction at the point of contact as determined from first principles. Author does not appear to be aware that the physical sig-

nificance of the Lagrangian parameters is precisely that they are the components of the reactions at the constraint.

Ewen M'Ewen, England

3766. Walker, G. E. L., Directional stability, *Auto. Engr.* **40,** 530, 533; 281–285, 370–376; Aug., Nov. 1950.

Paper discusses stability of automobiles, principally with regard to response of the car to gusts of wind and to turning. Principal factors determining stability are considered to be flexibility of the tires, type of suspension system, and aerodynamic forces. Aerodynamic forces are considered of increasing importance as the weight of the car is reduced, even though speed is simultaneously reduced somewhat. Principal measure of stability is "stability margin," defined as horizontal distance in percentage of wheelbase length between center of gravity of the car and center of reaction of road forces, latter axis being called "neutral steer line." Neutral steer line is a line such that a lateral force may be applied anywhere along it without producing any tendency for the car to rotate about the vertical axis. The car is considered directionally stable when the center of gravity is in front of the neutral steer line. Directional stability with steering fixed is considered desirable, but a large stability margin with concurrent slow response to control is considered undesirable.

Examples are given of comparative performance of unstreamlined, streamlined, and streamlined and stabilized cars. It is shown that the simply streamlined car may have a very rapid lateral divergence when exposed to a strong gust of wind, but that the addition of aerodynamic stabilization will produce a car with small sideslip and automatic recovery at a low rate that is controllable by the driver at his leisure.

It is further found that proper course stabilization of this sort, yielding small response to wind gusts from the side, will also benefit the turning or "cornering" behavior of the car.

Paper includes remarks on the effect of front- vs. rear-wheel drive on stability and concludes that sufficient data are now becoming available for confident design from the viewpoint of stability.

Herbert K. Weiss, USA

3767. Nadile, A., Influence of nonholonomic constraints on the displacement of the equilibrium configuration of a system (in Italian), *Riv. Mat. Univ. Parma* **1,** 393–399, 1950.

The displacement of an equilibrium configuration due to the introduction of holonomic constraints has been discussed by Levi-Civita ["Lezioni di Meccanica Razionale," vol. II, part I, p. 467]. An analogous discussion involving nonholonomic constraints is given here, including an investigation of stability.

D. C. Lewis, USA

3768. Lefèvre, P.-M., Interpretation rule of the generalized phase diagram (in French), *C. R. Acad. Sci. Paris* **232,** 5, 381–383, Jan. 1951.

A variant on the well-known stability criteria of Nyquist and Bode [AMR **4,** Rev. 1915] is restated in a form which materially reduces the numerical labor involved in application.

Herbert R. Lawrence, USA

3769. Malkin, I., On the stability of motion in the sense of Lyapunov, *Amer. math. Soc. Transl.* 41, 68 pp., 1951

Translation from *Mat. Sbornik* (*N.S.*) **3** (45), 47–100, 1938.

3770. Malkin, I., Some basic theorems of the theory of stability of motion in critical cases, *Amer. math. Soc. Transl.* 38, 50 pp., 1951.

Translation from *Prikl. Mat. Mekh.* **6,** 411–448, 1942.

3771. Ziegler, H., Approximation theory of the astatic pendulum (in German), *Vierteljahrsschr. naturforsch. Ges. Zürich* **95,** 173–197, Sept. 1950.

The astatic pendulum, consisting of short leaf spring placed vertically upward with rigid body at top, is frequently used for measuring gravitational field of earth, since its period T changes considerably with variations of the acceleration g, provided weight of body is taken slightly less than buckling load. Author solves integrodifferential equation of motion by process of successive approximations. He confirms the usual relation (Leiay) between δg and $\delta(1/T^2)$ and shows that for obtaining accuracies of 1 milligal, term with $\frac{1}{2}\delta(1/T^2)\}^2$ should also be considered. Furthermore, correction for nonharmonic character of the oscillation, depending upon admitted amplitudes, must be introduced. The better the pendulum is astaticized, the more important this correction will be. Author shows also that error, due to not exactly vertical clamping of spring at its root, will usually be negligible.

A. I. van de Vooren, Holland

3772. Gomory, R., and Richmond, D. E., Boundaries for the limit cycle of van der Pol's equation, *Quart. appl. Math.* **9,** 2, 205–209, July 1951.

Paper presents a simple means of establishing the outer and inner boundary curves for the limit cycle in the phase plane for the van der Pol equation $d^2x/dt^2 + \mu(x^2 - 1)\,dx/dt + x = 0$. Authors discuss their method and compare it with the more complex and somewhat less accurate method of LaSalle [*Quart. appl. Math.*, 1949]. Method requires the introduction of $r^2 = x^2 + y^2$ to transform van der Pol's equation in the phase plane to $r(dr/dx) = \mu(1 - x^2)(r^2 - x^2)^{1/2}$. The x^2 term under the radical is replaced by its least or greatest value in the internal, depending on the sign of the term $(1 - x^2)$ and on whether the inner or outer boundary is desired. The method applies when $x^2 - 1$ is replaced by other suitable functions $f(x)$.

Will J. Worley, USA

3773. Chudakov, E. A., Effect of the elasticity of tires on the stabilization of steering wheels of an automobile, *Izv. Akad. Nauk SSSR Otd. tekh. Nauk.* **8,** 1105–1111, Aug. 1950.

Experimental study of moments exerted by tire on front automobile wheel during steering. Lateral load at axle of rotating wheel with elastic tire displaces wheel center plane relative to ground contact surface. Result is a moment on the wheel, which may be stabilizing (tending to return wheel) or destabilizing, depending on operating conditions. Families of curves show this moment as a function of the angularity of wheel (amount of turn), tire pressure, load on wheel, and braking moment. Stabilizing moment rises with angularity, reaching a maximum at 4° to 6° angularity, then dropping off at about the same rate. Braking causes reduction in stabilizing moment which for braking moment of 35 kg-meters is shown to be destabilizing at 5.5° angularity under the test conditions.

Walter W. Soroka, USA

3774. Pinsker, I. Sh., Selection of a four-hinge linkage from a special atlas of curves (in Russian), *Akad. Nauk SSSR Trudi Sem. Teorii Mash. Mekh.* **9,** 33, 18–28, 3 plates, 1950.

This paper advocates a procedure of graphical interpolation for the determination of a four-hinge linkage nearly generating a given function as the angle ψ of one crank, the angle φ of the other crank being the independent variable. An atlas of (φ, ψ) curves (attached to the paper) is constructed for linkages whose bar lengths are in the ratios $i_1:i_2:i_3:i_4$, the i numbers running through the integral values 1, 2, 3, 4 and at least one of them being equal to 4. Certain five permutations of the i's lead to functions obtained by linear transformations (T) of φ and ψ

(reflection at the $\varphi = \psi$ line, at the φ axis, etc.). It is thus sufficient to consider at most four permutations of a given set of i's when constructing the atlas. The given curve is drawn to several scales on transparent paper (P), and each of these plots subjected to the five transformations (T). Then the paper is laid over the atlas, face up or down, and with the first bisectants (on the paper and the atlas) parallel with the same, or opposite senses. If satisfactory coincidence with a curve $i_1 i_2 i_3 i_4$ is observed, the manner of application of (P) determines the proper permutation of the i's defining the linkage, and other obvious data determine the initial points of the φ and ψ scales and their units. A complete numerical example is given (for $y = x^2$), and the error-estimate method indicated by a reference to a former paper [same *Trudi* **5**, no. 18, 34–83, 1948]. A. W. Wundheiler, USA

Gyroscopics, Governors, Servos

(See also Rev. 3768)

3775. Watzlawek, H., New problems of the artificial horizon and navigation on a gyro-physical basis (in German), *Öst. Ing.-Arch.* **4**, 1, 44–57, 1950.

Most instruments now used in aerial navigation to provide an artificial horizon employ a gyroscope with double gimbals which is "erected" to the vertical by some auxiliary device. The problem becomes one of applying corrections to this gyroscope so as to keep its axis close to the true vertical when it wanders off due to gimbal friction and other causes.

A way of erecting the main gyroscope is to use signals from a pendulum or some similar device which will be sensitive to the earth's attraction. However, any known device which indicates the vertical can be thrown off by accelerations along and normal to the flight path. In most present-day airplanes the flight path is in a straight line without distracting accelerations so much of the time that it suffices to have the erecting device inoperative when the aircraft is in yawing flight. Fortunately, there is in most airplanes a turn indicator (a gyroscope with one degree of freedom) for use in momentarily preventing erection to the false vertical which always accompanies curved flight and which, incidentally, has caused the death of many a pilot.

Author takes up the more difficult problem of erecting a gyroscope to the true vertical when there is little or no straight, unaccelerated flight. This might be the problem for a rocket ship when taking off in a turn or for a missile during complicated maneuvers. He gives a good mathematical review of the problem, along with the differential equations for gyroscopes in accelerated motion, and lists references to current literature. He then describes ways of detecting and influencing the processions of a gyroscope when it is oscillating about a desired position. The claim is made that it is possible to get indications of the true vertical, when an aircraft is in curved flight, with an accuracy of 1 min of angle.

Finally, there is a discussion of gyroscopic devices to indicate the course direction with a high degree of accuracy, and a list of basic patents relating to gyroscopic aids for aerial navigation. Reviewer feels that this article will interest all engineers concerned with the guidance and control of high-speed aircraft, even though it does not present the design of a finished instrument.
W. C. Johnson, Jr., USA

3776. Clarridge, R. E., An improved pneumatic control system, *Trans. Amer. Soc. mech. Engrs.* **73**, 3, 297–305, Apr. 1951.

Author discusses the response to large disturbances of industrial type pneumatic regulators, the primary large disturbance dealt with being process start-up. He presents a schematic diagram of a system referred to as a proposed controller. Differential equations are presented for comparing the dynamics of both systems. Author concludes that the proposed system can permit rapid response without overshoot during process start-up. No experimental data are presented. Harold Gold, USA

3777. Wolfe, W. A., Controller settings for optimum control, *Trans. Amer. Soc. mech. Engrs.* **73**, 4, 413–418 May 1951.

Paper deals with application of proportional plus reset controllers to processes having responses which can be approximated by one of two specific types, described in terms of the process output or "controlled variable" response to step function inputs. The output of the first type varies at a rate proportional to the input after a specific "dead time" or transportation lag time has elapsed. The output of the second type approaches a new value in exponential fashion after the dead time has elapsed; such a process is described as having "self regulation." Some processes may be so approximated and the conclusions of this paper would then help the designer select optimum constants.

Analysis is based on the transient response and employs the Laplace transformation. "Optimum" settings determined are those which cause a minimum time integral of the error following a disturbance and which produce sufficient response damping. Mathematical relations describing optimum settings are not explicit and graphical evaluation is generally necessary.

Importance of adding a "reset" feature to a proportional controller is demonstrated. Errors caused by disturbances then ultimately approach zero. A criterion is given for selecting constants which provide minimum error integral for a specified damping ratio. With such constants, the output generally overshoots the "set point" or "reference input." A modified criterion is given for applications where such overshoots are undesirable. It is found that satisfactory control is easier to achieve for processes of the second type having "self-regulation."

Reviewer believes that the results will be of value in the design of some process control systems which can be approximated as described. The transient approach used would be too unwieldy for a more complicated process, for which recourse to analog-computer techniques and the frequency-response method of analysis [AMR **3**, Rev. 11] would be necessary. Reviewer finds that conclusions reached here are consistent with those obtained by the frequency-response method.

Reviewer questions minimizing the error integral as a basis for selecting optimum constants. Other studies [AMR **2**, Rev. 1359] have indicated that the root mean square error is a superior basis. Nevertheless, the criterion used in the paper probably is adequate for a number of applications.
Ralph J. Kochenburger, USA

3778. Profos, P., The dynamic behaviour of the control stage in pressure control (in German), *Schweiz. Arch.* **17**, 4, 114–119, Apr. 1951.

Author deals with pressure-governing and derives equations for the controlled line. The frequency-response diagrams for most important cases are arranged in a table which shows the behavior of these different layouts. It is evident that some arrangements cannot be obtained without back-coupling.
Miroslav Nechleba, Czechoslovakia

3779. Himmler, C. R., Hydraulic control (La commande hydraulique) (in French), Paris, Dunod, 1950, vi + 316 pp. Frs. 1780.

Author, for 18 years a member of the Siemens and Halske Laboratories and of the German Institute for Aeronautical Research, is the author of numerous articles and patents. He be-

lieves that hydraulic (oil) means of control are superior to electronic, particularly in regard to weight, for distances less than 50 ft. Chapters I to III of the book are about physical laws of flow; Chapters IV to X about hydraulic pumps, motors, and accessories; Chapters XI to XVIII about hydraulic control, with specific applications to fire control, automobiles, regulators, and airplane installations. Book contains an enormous amount of information; the 237 figures in particular are, for the most part, either experimental curves giving relations between physical variables, or else detailed drawings of actual motors or installations. Unfortunately, the information is so condensed that a beginner would probably be unable to understand it; but no one with some previous knowledge of hydraulic control should miss this book (even if he should have but little command of French, since the many figures tell most of the story). P. LeCorbeiller, USA

3780. Nikol'skiĭ, G. N., On a problem of indirect regulation (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **4**, 2, 113–132, 1948.

Paper is concerned with stability conditions for a closed-cycle control mechanism containing a servomotor of nonlinear characteristic. A typical problem as considered is described by the equations (1) $P(D)y = Q(D)x$, (2) $Dx = \varphi(x + y)$, where x measures the motion of the servomotor, $x + y$ the activating impulse; P, Q are polynomials with constant coefficients in $D = d/dt$, and φ is the nonlinear characteristic of the servomotor, for which $\varphi(0) = 0$, $x\varphi(x) \leqq 0$. It is assumed that the servomotor when activated by a harmonic impulse carries out, after a transient, a periodic motion $x = x(t, \tau)$ of period τ and of amplitude $x(\tau, \tau)$. If $y(t, \tau)$ is the periodic solution of (1) for $x = x(t, \tau)$, then the stability condition, derived by the author from physical considerations, is $x(\tau, \tau) + y(\tau, \tau) > 0$ for all $\tau > 0$.
Courtesy of Mathematical Reviews M. Golomb, USA

3781. Newton, G. C., Jr., Comparison of hydraulic and electric servomotors, *Proc. nat. Conf. Industr. Hyd.* **3**, 64–86, Mar. 1950

Author has considered in detail the problem of selecting a proper motor for use in a linear servo control system which has to position a specified mechanism in agreement with a given input function. Assume that the servomotor is characterized by a certain inertia J_M and that it can develop a certain peak torque T_{MP}, and a peak velocity V_{MP}. Further assume that the motor is coupled to the load through an ideal gear train (without inertia, friction, or backlash) having a reduction ratio R, and that the load has an inertia J_L and can demand a peak torque T_{LP}, a peak velocity V_{LP}, or a peak acceleration a_{LP}. Although peak velocities and accelerations may not be truly independent, author considers that the worst possible condition is encountered when these two quantities reach their peak values simultaneously. Define two quantities, called time constants, $\tau_{MP} = V_{MP}T_{MP}/(T^2_{MP}/J_M)$ and $\tau_{LP} = V_{LP}/a_{LP}$. If P_{LP} and P_{MP} are, respectively, the peak power absorbed by the load and the peak power generated by the motor, author establishes the following inequalities

$$P_{MP}/P_{LP} \geqslant 1/(1 - \tau_{MP}/\tau_{LP}) \text{ if } \tau_{MP}/\tau_{LP} \leqslant 1/2$$

and

$$P_{MP}/P_{LP} \geqslant 4\tau_{MP}/\tau_{LP} \text{ if } \tau_{MP}/\tau_{LP} \geqslant 1/2$$

on the basis of which a proper motor can be selected.

Author has included a discussion of geometrically similar lines of motors and compares the performance of hydraulic and electric motors in terms of their time constants. Considerable engineering information is tabulated and a rational procedure for selecting a servomotor to satisfy specified load requirements is outlined. Finally, it is shown that neglect of inertia effects in a rationally designed gear train introduces only small errors in the analysis. Horace M. Trent, USA

Vibrations, Balancing

(See also Revs. 3748, 3772, 3828, 3967)

3782. Arnold, R. N., and Warburton, G. B., Flexural vibrations of the walls of thin cylindrical shells having freely supported ends, *Proc. roy. Soc. Lond. Ser. A*, **197**, 238–256, June 1949.

Paper deals with the general theory of vibration of thin cylinders and a special investigation is made of the type of vibration associated with bells. The cylinders are supported in such a manner that the ends remain circular without directional restraint being imposed. The frequency equation which is derived by the energy method is based on strain relations. Results are given for cylinders of various lengths, each with the same thickness-diameter ratio, and also for a very thin cylinder in which the simpler modes of vibration occur in the higher frequency range. It is shown that there are three possible natural frequencies for a particular modal pattern, two of these normally occurring beyond the aural range.

From authors' summary by C. J. Bernhardt, Norway

3783. Kouvelites, J. S., Free longitudinal vibration of a prolate ellipsoid, clamped centrally, *Quart. appl. Math.* **9**, 1, 105–108, Apr. 1951.

Author extends the well-known solution for the free longitudinal vibrations of constant cross-section bars to the problem given by the title. The new equation of motion is then

$$\frac{\partial^2\xi}{\partial x^2} - \frac{2x}{a^2 - x^2}\frac{\partial\xi}{\partial x} = \frac{\rho}{k}\frac{\partial^2\xi}{\partial t^2}$$

where ξ is the longitudinal displacement, ρ the volume density, k the modulus of elasticity, and $2a$ the length of the bar. After separating the variables by writing $\xi = X(x)T(t)$, the equation in X is solved by means of a power series. Expressions are given for the first three resonant frequencies. H. D. Conway, USA

3784. Klotter, K., The bending vibrations of a beam with arbitrary boundary conditions under a periodic axial force (in German), *Ing.-Arch.* **18**, 6, 363–369, 1950.

Author considers the stability of a beam acted upon by a periodic axial force $P_0 + S\cos\Omega t$, where P_0, S, and Ω are constants, and t is time. Case when the beam is simply supported at both ends has been treated by Lubkin and Stoker [*Quart. appl. Math.* **1**, 215–236, 1943]. This case leads to Mathieu's equation, and the stability of the system is deduced from known properties of Mathieu functions. In present paper a variety of end conditions are considered, including built-in conditions. The transverse deflection $y(x, t)$ is determined as an infinite sum of products of Mathieu functions of t and trigonometric and hyperbolic functions of x, with a quadruply infinite set of constant coefficients to be determined so that four functions of t vanish identically. The actual determination of these constants is not carried out, but an approximate determination of them is indicated. Known properties of Mathieu functions again permit a determination of the stability of the system. G. E. Hay, USA

3785. Litwiniszyn, J., A certain boundary problem of a vibrating string, *Arch. Mech. stos.* **2**, 2 75–88, 1950.

Paper deals with the problem of a longitudinally or transversally

vibrating string the length of which changes during vibration according to a given function $\lambda(t)$ of time t (e.g., hoisting rope). State of string at both ends $s = 0$ and $s = \lambda$ is a given function of t. Problem is solved by means of Riemann's method of integration. The region in the s–t plane, to which Green's theorem has to be applied, is bounded by the two characteristics intersecting in the point under consideration together with the s and t-axes and the curve $s = \lambda(t)$.

For the special case of uniformly increasing or decreasing length of the string, $\lambda(t) = l + \beta t$, it is shown that by a suitable choice of $|\beta| < a$ the strain in the string may assume arbitrarily large values. (a is velocity of propagation of waves in the string.) Heinz Parkus, Austria

3786. Radok, J. M., Vibrations of a swept box, *Coll. Aero. Cranfield Rep.* 47, 12 pp., Apr. 1951.

Using Hamilton's principle to deduce the equations of dynamic equilibrium of a swept box beam of uniform cross section with ribs and stringers but without additional masses and with shear neglected, author proceeds to show how the integrodifferential equations may be reduced, for the case of cantilever fixity, to a homogeneous system of integral equations of the Fredholm type. A method of numerical solution for these equations is indicated.
James B. Duke, USA

3787. Rauch, L. L., Oscillation of a third-order nonlinear autonomous system, *Contr. theory Nonlinear Oscill.*, study 20, 39–88, 1950.

Author discusses oscillations that arise in a nonlinear system involving a vacuum tube, and which is of such a nature that it can be dealt with by means of a differential equation of third order not involving the time explicitly. The main object of the paper is then to discuss the solution curves in the phase space, which is three-dimensional. A torus is constructed in this space in such a way that the field vectors at the boundary of the torus defined by the differential equation all point toward the interior of the torus—at least when the physical parameters lie within certain specified ranges. It is shown that the solution curves all circulate the entire way around the torus. From these facts follows, by use of Brouwer's fixed-point theorem, the existence of at least one closed-solution curve, and hence the existence of a periodic solution. The uniqueness and stability of this periodic solution are difficult to discuss, but author shows that every solution, with two possible exceptions, eventually enters the torus and hence stays within a certain bounded distance of the periodic solution. The paper has much in common with a paper by Friedrichs [K. O. Friedrichs: "On nonlinear vibrations of third order," in Studies in Nonlinear Vibration Theory, Institute for Mathematics and Mechanics, New York University, 1946] in which a system of third order occurs, and a torus with like properties is constructed in a phase space; but the author's differential equation is different and he interests himself in a number of questions not treated by Friedrichs. J. J. Stoker, USA

3788. Chekmarev, A. I., Nonlinear oscillations of antivibrators (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **5**, 1, 140–157, 1948.

Author treats the problem of torsional damping by means of pendulums. It is shown that the problem is expressible in terms of two nonlinear differential equations of the second order, one of which has an external periodic excitation. In these equations the coefficients of the second derivatives are trigonometric functions of dependent variables. Ritz's variational method is used to obtain the expression for the forced oscillation. This reduces the problem to a transcendental equation involving Bessel's functions J_0 and J_1 as well as parameters from which the equivalent moment of inertia of the antivibrator is computed. It is shown that this quantity is nearly constant for small oscillations and is a function of the amplitude for larger ones. In the latter case the method of continuous fractions, attributed to V. P. Terskikh, is applied. The method itself is not outlined in the paper.
N. Minorsky, France

3789. Mendelson, A., and Gendler, S., Analytical determination of coupled bending-torsion vibrations of cantilever beams by means of station functions, *Nat. adv. Comm. Aero. Rep.* 1005, 20 pp., 1951.

See AMR **4**, Rev. 1459.

3790. Geiger, J., On horizontal vibrations in steam-turbine foundations (in German), *Schweiz. Bauztg.* **68**, 31, 424–426, Aug. 1950.

For turbo-sets with reduction gears between the turbine and electric generator, for which the speed of the generator is low, the frequency of the imposed oscillation may be near the natural frequencies of horizontal vibration of the portal foundation. Author calculates natural frequencies by assuming the table plate is absolutely rigid, and gives approximate determination of the spring resilients for a symmetrical foundation plate elastic in the transverse direction. Article gives useful basis for practical calculations. Pavel Kohn, Czechoslovakia

3791. Geiger, J., On vertical vibrations in steam-turbine foundations (in German), *Schweiz. Bauztg.* **69**, 21, 287–290, May 1951.

Reduction of pillar mass, bending elasticity of foundation sole, elasticity of the ground, torsional deformation of the longitudinal beams, edge rigidity between pillars, and transverse beams of portal foundations are discussed concisely, and practical data are given. Reviewer believes that measured natural frequencies are lower than the calculated due to nonlinearity of the ground elasticity. Pavel Kohn, Czechoslovakia

Wave Motion, Impact

(See also Revs. 3827, 3915, 4007)

3792. Takeuchi, M., On the lateral impact of beams, *Memo. Fac. Sci. Engng. Waseda Univ., Tokyo*, no. 14, 114–116, 1950.

By means of an elastic string a mass is connected with the midpoint of a beam with simply supported ends. An impulse is given to the mass. The tensile force in the string, the deflection of the beam, and the motion of the mass are studied.

The system is approximated by a simple vibrating system of two degrees of freedom and, for a string that is very stiff, by a system of one degree of freedom. An example shows relatively large errors on the safe side compared with the step-by-step integration method of the exact equations.
W. L. Esmeijer, Holland

3793. Ursell, F., Trapping modes in the theory of surface waves, *Proc. Camb. phil. Soc.* **47**, part 2, 347–358, Apr. 1951.

It is shown that a mass of fluid bounded by fixed surfaces and by a free surface of infinite extent may be capable of vibrating under gravity in a mode (called a trapping mode) containing finite total energy. Trapping modes appear to be peculiar to the theory of surface waves; it is known that there are no trapping modes in the theory of sound. Two trapping modes are constructed: (1) A mode on a sloping beach in a semi-infinite canal of finite width, and (2) a mode near a submerged circular cylinder in an infinite

canal of finite width. The existence of trapping modes shows that, in general, a radiation condition for the waves at infinity is insufficient for uniqueness.

From author's summary by J. J. Gilvarry, USA

3794. **Zaĭtsev, L. P., and Zvolinskiĭ, N. V., Investigation of the head wave arising at the boundary between two elastic liquids** (in Russian), *Izv. Akad. Nauk SSSR Ser. Geograf. Geofiz.* 15, 20–39, 1951.

Authors call a "head" wave that which is known in seismology as a "refracted" wave. They consider the propagation of a disturbance in two dimensions in an infinite medium composed of two liquids, applying the method developed by Smirnov and Sobolev [cf. Sobolev, chap. 12 of Russian edition of Frank and von Mises, "Die Differential- und Integralgleichungen der Mechanik und Physik," ONTI, 1937]. The solutions of the wave equation are represented by the real parts of an arbitrary function of a complex variable θ. This variable is given by the equation $t - \theta x + y(a^{-2} - \theta^2)^{1/2} + p(\theta) = 0$, where p is an arbitrary function. On assuming a certain form of velocity potential created by a point (line) source, the boundary conditions are satisfied by a choice of other potentials. The corresponding displacements for the incident, reflected, and "head" wave are derived in an explicit form. Some conditions seem to be given in an incorrect form.

Courtesy of Mathematical Reviews W. S. Jardetzky, USA

Elasticity Theory

(See also Revs. 3757, 3831, 3862)

3795. **Novozhilov, V. V., Foundations of the nonlinear theory of elasticity [Osnovi nelineĭnoĭ teorii uprugosti]**, Moscow-Leningrad, Gosud. Izat. Tekhn.-Teor. Lit., 1948, 211 pp.

This book makes a twofold contribution to the available literature on elasticity. For, in addition to presenting a penetrating account of modern thinking on the nonlinear theory, it is also a clear account of the fundamentals of the modern mathematical theory as a whole. In fact, one gets the impression that the author's rejection of the usual linearizing assumptions in the derivation of the fundamental relations enhances (rather than encumbers) the reader's comprehension of the geometric and physical assumptions of the theory. In its use of mathematical tools the book falls between the engineering texts and, say, the work of Sokolnikoff and Muskhelishvili. The use of tensors is avoided; however, the usually cumbersome component notation is employed with great skill. Except for the language difficulty, the book would make an excellent companion piece to Prescott ["Applied Elasticity," London, Longmans, Green, 1924] for use by first- and second-year graduate students. The present book supplies the physical insight into the theory which is lacking in Prescott, but without the wealth of examples found in the latter work.

The analyses of strain, stress, and of the equilibrium equations are presented in full generality for homogeneous isotropic bodies, and the classical relations are obtained by a two-stage modification of the resulting expression. It is shown that the classical formulation contains two assumptions to the effect that (a) the strains and the angles of rotation of the body are small compared to unity, and (b) the products of the angles of rotation are small compared to certain corresponding components of strain. The specific strain energy of a body is represented as a series expansion in terms of the strain invariants, and the stress-strain relations are given in terms of the coefficients of this expansion. The specific linearity assumptions underlying Hooke's law are thus made explicit. On the basis of two experimental observations of the behavior of the stress invariants, the same formulation (originally derived for conservative forces) is shown to yield the Hencky stress-strain equations for loading in plastic bodies. Two types of nonlinearity are shown to enter the problems of deformation of elastic bodies. The geometrical nonlinearity results when the angles of rotation of the body and the strains are no longer negligible compared to unity. The physical nonlinearity results when the strains are no longer negligible compared to certain physical constants of the material (proportional limits).

The chapter headings are as follows: (I) Geometry of strain; (II) Equilibrium of a volume element; (III) Strain energy, boundary conditions, stress-strain law; (IV) Formulation of the elastic (boundary-value) problem in terms of stresses; (V) The problem of elastic stability; (VI) Deformation of elastic bodies.

Chapter VI presents applications of the nonlinear theory to the following cases: (a) bending of thin plates and shells; (b) bending and torsion of rods. An extensive bibliography is appended.

Courtesy of Mathematical Reviews H. I. Ansoff, USA

3796. **Parker, H., Simplified mechanics and strength of materials**, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1951, xiv + 275 pp. $4.

3797. **Hencky, H., New methods in mechanics of materials [Neuere verfahren in der festigkeitslehre]** München, Verlag R. Oldenbourg, 1951, 72 pp.

This short book is the introductory part of an extensive work which was completely destroyed by fire in 1943. The "new methods" discussed and strongly recommended are: (a) Tensor methods and subscript notation; (b) virtual work employed directly on the three-dimensional equations of elasticity to arrive at plate, beam, and shell equations and approximate solutions to practical problems.

Topics discussed include differential geometry, statics, kinematics, invariants and their value in elasticity, plasticity, and failure considerations. Of special interest is the recommendation that the second invariant of the stress deviation (square of octahedral shearing stress) be plotted against the first invariant (sum of principal stresses). Also, many detailed derivations and applications show, it is claimed, the value and the use of virtual work directly instead of the variational principles of elasticity for plates, beams, and shells when geometrical assumptions and approximations are made. In this section of the book the equations are formulated in orthogonal curvilinear coordinates without tensor notation.

Many typographical errors occur, and at times rather sweeping claims are made and difficult points passed over. These difficulties are undoubtedly due to the circumstances under which the book was published. D. C. Drucker, USA

3798. **Radenković, D., A solution of the two-dimensional problem in the theory of elasticity** (in Russian), *Publ. math. Inst. math. Acad. Serbe Sci.* **3**, 127–136, 1950.

After summarizing the general method given by Love [*J. Lond. math. Soc.* **3**, 144–156, 1928] for the determination of a biharmonic function holomorphic in a given plane region and satisfying specified conditions on the boundary of the region, author goes on to give the details of the analysis for the two-dimensional problem of a rectangular elastic plate under given edge loads. The method involves, in the first place, the determination of a harmonic function $\Phi(\theta)$ which reduces to specified values θ on the boundary. The boundary condition for the biharmonic stress function χ can then be expressed in the form $\chi = 0$; $\partial\chi/\partial\nu = -\partial\Phi(\theta)/\partial\nu$. By transforming the region within the

rectangle to the region within a unit circle $\rho = 1$, such a harmonic function is determined as a Fourier series of cosines. The biharmonic function χ is then expressed as a series of separate biharmonic functions $\chi_n^{(n)}$ in the form $\chi = \sum_{n=1}^{\infty} A_n \chi_n^{(n)}$ which can also be determined as Fourier series of cosines. By comparing the values of $-\partial\Phi(\theta)/\partial\nu$ and $\partial\chi/\partial\nu$ on the boundary, the coefficients A_n may be found. Having determined the biharmonic function χ, the stresses $\sigma_x = \partial^2\chi/\partial y^2$, $\sigma_y = \partial^2\chi/\partial x^2$, $\sigma_{xy} = -\partial^2\chi/\partial x\partial y$ can be evaluated, and author has tabulated the required numerical values of the derivatives $\partial\chi_n^{(n)}/\partial\rho$ and $\partial^2\chi_n^{(n)}/\partial\rho^2$ as far as $n = 6$, for points along the axis of y. These enable him to evaluate σ_x along the axis of y which he has graphed, and his results are in close agreement with those of other authors.
Courtesy of Mathematical Reviews R. M. Morris, USA

3799. Grioli, G., The structure of Airy's stress function in multiply connected regions, *Nat. adv. Comm. Aero. tech. Memo.* 1290, 34 pp., July 1951.
See AMR **2**, Rev. 1108.

3800. Grinberg, G. A., On the solution of the plane problem of the theory of elasticity and on the flexure of a thin plate with clamped edges (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)*, **76**, 5, 661–664, Feb. 1951.

Author solves the boundary problem $\nabla^4\psi = \Phi$ in f; $\psi|_s = r(s)$; $\partial\psi/\partial n|_s = q(s)$, where s is the boundary of the domain f, and $r(s)$, $q(s)$, and Φ are given. The problem is split into the two separate problems, $\nabla^2\theta = \Phi$; $\nabla^2\psi = \theta - F$, $\nabla^2 F = 0$. By means of Gauss' formula, F can be expressed as a series of orthonormal functions ψ_n, derived from an arbitrary but complete set of harmonic functions φ_n. Thus, author succeeds in splitting the integration of the initial fourth-order equation into the integration of two second-order equations, even if the boundary conditions for these two equations are not identical. This is possible because F is expressed in terms of the ψ_n and of $r(s)$ and $q(s)$, i.e., contains the boundary conditions.

As a first example, author considers a clamped square plate under uniformly distributed load and compares the values for the maximum deflection and maximum bending moment, obtained by using five terms in the series in the ψ_n, with those given in Timoshenko's "Theory of plates and shells." The error is within the accuracy of engineering computations.

A second example treats the plane stress problem of an elastic square under a tension, uniformly and symmetrically distributed over a portion c of the edge a. For various ratios c/a the error between the numerical values of the sum of principal stresses at the center, obtained by the ψ_n of the previous problem, and by other methods, is again well within engineering accuracy.

Finally, a theorem is derived by which deflections of a clamped plate can be determined from those of a simply supported plate, once F is known. George Herrman, USA

3801. Arzhanĭkh, I. S., On the theory of integration of dynamical equations of an isotropic elastic body (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **73**, 41–44, 1950.

Present paper is devoted to various questions concerning the integration of the dynamical equations of an isotropic elastic body: (1) $\partial^2 u/\partial t^2 - \alpha$ grad div $u - \beta$ rot rot u, where the vector-valued function u is the displacement, and $\alpha = (\lambda + 2\mu)/\rho$, $\beta = \mu/\rho$, where λ and μ are Lamé's constants of elasticity, and ρ is the density of the body. Author first gives certain general representations of solutions of (1) in terms of arbitrary functions, which extend known results in the static case (when $\partial^2 u/\partial t^2$ is replaced by zero). Let S denote a smooth surface, and Q denote either its interior or its exterior. If Q is the exterior of S then u is required to be "regular at infinity," in addition to satisfying (1). Using a lemma derived previously, author derives integral equations which are equivalent to the solution of certain free and forced vibration problems for the body Q.
Courtesy of Mathematical Reviews J. B. Diaz, USA

3802. Mettler, E., A general theory of the stability of forced vibrations of elastic bodies (in German), *Ing.-Arch.* **17**, 6, 418–449, 1949.

This paper, which continues a previous paper by the same author [AMR **2**, Rev. 29] employs the same basic analysis, it now being assumed that the applied body and surface forces comprise static terms together with dynamic terms that are simple harmonic in time. Attention is given only to the development of analysis to yield approximate solutions of the general problem, and, as author states, there is no attempt here to rigorously justify this analysis. As an example, detailed analysis (including numerical results) is given of the stability of a rectangular bar under end-bending moments that involve static and dynamic terms. H. G. Hopkins, England

3803. Krzywoblocki, M. Z., On the so-called principle of least work method, *Öst. Ing.-Arch.* **5**, 1, 81–96, Feb. 1951.

Paper extends the principle of Castigliano to cover case of continuously distributed stresses. The least-work method is widely used in engineering practice to solve this type of problem. Author shows by several examples how conventional application of method (by equating to zero first partial derivatives of strain energy with respect to unknown coefficients) can give incorrect results although convergency is good. Reason for discrepancy is that the chosen functions do not always give even approximately the absolute minimum of strain energy. Author therefore proposes following generalized principle: "When applying Castigliano's theorem we must be assured a priori that the redundant generalized forces or unknown stress distributions are expressed by functions among which there is one, and only one, function which gives a smaller value of the total strain energy than any other gives, having first taken the given values of the boundary."

Reference is made to a paper by Pöschl ["Über die Minimalprincipe der Elastizitätstheorie," *Bauingenieur* **17**, 17/18, 1936], which points out variational nature of problem.

No example is given showing concrete application of generalized principle. Sverker Sjoström, Sweden

3804. Shapiro, G. S., Elastic equilibrium of a paraboloid of rotation (in Russian), *Prikl. Mat. Mekh.* **14**, 6, 672–673, Nov.-Dec. 1950.

Love's stress function ϕ for a paraboloid of rotation can be expressed as an infinite series containing Legendre's functions of the first and second kind, if the axially symmetric load on its surface can be presented in the form of a polynomial of the coordinate along its axis. For a paraboloid of the second-order with the equation $z = a\,r^2$ loaded with a pressure $\gamma \cdot z$, the stress function can be expressed as a sum of four terms and the constants in it are determined by stress boundary conditions on its surface. The definitive stresses are: $\sigma_r = \sigma_\theta = \gamma \cdot (z - 1.8a)$, $\sigma_z = \gamma \cdot z/2$, $\tau_{rz} = -\gamma \cdot r/4$.

Superposing a uniform pressure $-\gamma \cdot z$ in all directions, author obtains the stress distribution in a paraboloid under the influence of its own weight. Anton Kuhelj, Yugoslavia

3805. Shtaerman, I. Ya., The contact problem in the theory of elasticity (in Russian), Gostekhizdat, 1949, 270 pp.

This book represents results of work by the author, and other

Russian scientists, on the contact problem. In the first of four chapters, one third of book, author shows solution of fundamental integral equation of the plane contact problem for various cases.

In the second chapter he derives the fundamental equation of the plane contact problem, followed by application to two elastic bodies with one and many segments of contact. Also discussed are pressures between two elastic bodies with periodic contacts, rigid body acting on an elastic one, and contact pressures between elastic bodies considering friction.

In chapter three, author shows solution of fundamental integral equation of axisymmetrical contact problem. He considers problems of finding pressure between bodies with surfaces of revolution, and pressure of circular cylindrical die on elastic semi-infinite body.

The last chapter is devoted to the general case of the contact problem. Here author finds the potential of elliptical disk, pressure between elliptical die and semi-infinite body, and compression between two elastic bodies initially touching at a point.

The appendix contains a discussion on: "The reduction of some elliptic integrals to the canonical form," and "On approximate solution of some integral equations of the contact problem."

Book is an important mathematical contribution to the subject.

S. Sergev, USA

3806. Hannah, M., Contact stress and deformation in a thin elastic layer, *Quart. J. Mech. appl. Math.* **4,** part I, 94–105, Mar. 1951.

Author considers problem of rollers used in drafting textile fibers, the lower one made of steel and the upper with a metal center and a fairly thin elastic cover. The length over which the rollers are in contact and the pressure distribution over this length are determined as functions of the material and thickness of the cover, of the load on the rollers, and of the roller size. The system is considered as one of generalized plane stress in an elastic layer, with given displacement conditions at the interface between metal and cover, and subject to pressure by a body of given shape on its free face. Analysis is given for flat layer with isolated force on one face and no horizontal or vertical displacement on other face. Author states that this analysis can be applied to an annular layer if the layer thickness is small relative to the diameter and, on this basis, presents solution of problem. As layer thickness approaches infinity, solution approaches Hertz solution.

For thin layers, author finds that, next to roller diameter and elastic modulus, layer thickness is the most important factor determining the relation between loading and deformation. As the elastic layer becomes thinner, a greater load is necessary to produce the same contact length The pressure distribution over the contact zone is practically insensitive to layer thickness. Numerical results are presented for values of Poisson's ratio of 0 and $1/2$ and for the cases where the inner boundary is fixed and where it is free to slip along the interface. S. Levy, USA

3807. Arienti, R., Thermoelastic stresses in a freely extensible flat plate pervaded by a battery of heating tubes (in Italian), *Termotecnica* **4,** 10, 484–493, Oct. 1950.

Author considers the problem of a freely supported semi-infinite plate $0 \leqq x \leqq s$, $|y| \leqq l/z$, having a uniformly distributed and steady source of heat along an interior plane x = constant, for the case of thermal equilibrium. Employing standard engineering approximations, he treats, in the first place, the case $l = \infty$ and obtains equations connecting the two temperature gradients in the plate with its (average) curvature and with the σ_y stresses on both sides of the plane x = constant mentioned previously. In the second place, he considers the case of finite l and obtains refinements of the previous result by the use of a well-known variational method.

Author believes that the results should be of practical interest in the case of structures internally heated with a network of heating tubes.

Reviewer feels that much more convincing results would have been obtained by using the rigorous procedures of the mathematical theory of elasticity, notwithstanding the apparent complexity of the problem when tackled by less powerful tools.

Albert W. Saenz, USA

3808. Pólya, G., and Weinstein, A., On the torsional rigidity of multiply connected cross-sections, *Ann. Math.* (2), **52,** 154–163, 1950.

Consider a cylinder whose cross section is multiply connected, i.e., is bounded by an outer curve and by several inner curves which include holes. The main result of the present paper is the following general theorem: Of all multiply connected cross sections with a given area and with given joint area of the holes, the ring bounded by two concentric circles has the maximum torsional rigidity. The theorem generalizes a celebrated proposition due to Saint-Venant, which corresponds to the extreme case when there are no holes. Saint-Venant's statement has been proved only recently by the method of symmetrization [Pólya and Szegö, *Amer. J. Math.* **67,** 1–32, 1945; Pólya, *Quart. appl. Math.* **6,** 267–277, 1948]. The proof of the main result is also based on the method of symmetrization, and it is preceded by a careful survey of the whole problem of torsion for multiply connected domains. In order to illustrate the theorem, the upper bound $(q^4 - p^4)/2\pi$ is obtained for the torsional rigidity S of a frame whose inner boundary is a square of side p and whose outer boundary is a concentric square of side q. This upper bound for S is slightly better than the upper bound $\frac{1}{6}(q^4 - p^4)$ (the polar moment of inertia of the frame) which was given by Diaz and Weinstein [*Amer. J. Math.* **70,** 107–116, 1948], who gave both upper and lower bounds for S.

Courtesy of Mathematical Reviews J. B. Diaz, USA

3809. Rivlin, R. S., Mechanics of large elastic deformations with special reference to rubber, *Nature* **167,** 4250, 590–591, Apr. 1951.

Author introduces the three "principal extension ratios" $\lambda_1, \lambda_2, \lambda_3$, defined as one plus the strains in the principal directions. The free energy W is considered as a function of three independent functions of λ_1, λ_2, and λ_3, essentially strain invariants I_1, I_2, and I_3. The load-deformation function for any given isotropic, homogeneous, and elastic material is uniquely determined from corresponding function W. The assumption is made that the material is incompressible, making I_3 constant. Dependence of W on I_1 and I_2 is established experimentally, using a sheet of vulcanized rubber. From W more complex types of deformation may be calculated, and author shows as an example the case of simple torsion of a right-circular cylinder. Results show agreement with the Poynting experiment: residual tension stresses.

Frithiof I. Niordson, Sweden

3810. Föppl, L., The exact solution for rolling friction [Die strenge Lösung für die rollende Reibung], München, Leibniz-Verlag, 1947, 44 pp., 13 figs. DM 6.

Author attacks afresh the problem of distribution of compressive stresses and shear stresses in the surface of contact between wheel and rail (supposedly of equal material) and gives an analytically satisfactory solution.

First, he calculates the distribution of shear stresses resulting from a tangential force, assuming that sliding occurs nowhere in

the surface of contact, and he shows that under this assumption the stresses on the rear side become unbounded (i.e., that sliding cannot be totally prevented). Next, he calculates the distribution of shear stresses, assuming that there is sliding over the whole of the surface of contact.

After these two limiting cases are dealt with, author supposes that the surface of contact is divided into two areas, a front area, where no sliding occurs, and a rear area, where sliding does occur. In order to procure a stress distribution which satisfies all necessary boundary conditions, three stress distributions of the above-mentioned kind are superimposed; they contain seven free parameters. Six of them can be determined by considering the shear stresses alone; for determining the last one, the compressive stress distribution has to be considered also.

From the distribution of the compressive stresses there follows the so-called "principal equation" of the theory, containing the moment M, the tangential force T, the normal force N, the ratio α between the sliding and nonsliding area, and the coefficient of friction μ_0. For given values of M, T, N, this equation determines the ratio α. In discussing his principal equation, author finds that $\alpha_0 = 0.64$ is a significant value, and that this value, α_0, will be present whenever T is different from zero. The maximum value for the ratio $\mu = T/N$ is found to be $\mu = 1.136\,\mu_0$.

The amount of sliding resulting from $\alpha_0 = 0.64$ is compared with observations by G. Sachs (1924) and good agreement is reported. K. Klotter, USA

Experimental Stress Analysis

(See also Rev. 3862)

3811. Drucker, D. C., The method of oblique incidence in photoelasticity, *Proc. Soc. exp. Stress Anal.* **8,** 1, 51–66, 1950.

This report considers briefly some of the available methods for the analysis of three-dimensional states of stress, in particular, the photoelastic "freezing" technique. The present state of the art is discussed and it is pointed out that all previous analyses have employed either sections of symmetry or a knowledge of the type of stress distribution, as, for example, in the case of torsion or bending. If photoelasticity is to be truly useful for three-dimensional problems, methods should be perfected which will enable complete analysis of the state of stress at each point without the necessity for choosing particular planes or directions. However, the first step is to develop proper methods for simpler cases.

The problems of accuracy are considered for the principal plane and the two-dimensional case which will serve as the pilot tests for more general means of analyzing slices cut from "frozen" models. The procedure using a single oblique photograph is compared with a more recent proposal of double oblique incidence. It is shown that when the principal stresses in the plane have the same sign, the formulas for single oblique incidence have higher accuracy than those proposed for double incidence. This indicates the necessity for modifying the latter when studying principal slices.

An alternative double oblique-incidence procedure is presented in which retardation values alone determine accurately the magnitude of the principal stress differences.

From author's summary

3812. Kammerer, M. A., Three-dimensional photoelasticity (in French), *Ann. Ponts Chauss.* **121,** 1, 71–103, Jan.-Feb. 1951.

After a review of the distribution of strains in elastic bodies, this memorandum is devoted to a discussion of the principal experimental procedures employed in three-dimensional photoelasticity. Use of stress optically sensitive inclusions, "freezing" of strains, and the scattered-light technique are described.

Most important remarks concern experimental errors in various procedures. Author concludes that while the ideal material is still to be developed, existing plastics may be employed, provided extreme accuracy is not required. Of interest are author's descriptions of the photoelastic apparatus constructed especially for the laboratories of the French National Railways.

L. E. Goodman, USA

3813. Brown, A. F. C., Measurement of the curvature of stress trajectories in photoelastic models, *Brit. J. appl. Phys.* 2, 5, 138–140, May 1951.

The use of Jessop's extension of the Lamé-Maxwell equations for separating the principal stresses in a plane of symmetry of a three-dimensional photoelastic problem depends for its accuracy on a precise knowledge of the curvature of the stress trajectories crossing the line along which the integration process is carried out. A method is described for measuring these curvatures by obtaining accurately (to about $^1/_{20}$ degree) the isoclinic angle at the extremities of the chord, in the trajectory of which the curvature is to be measured. An application is made to photoelastic slices obtained from the reduced section of a round bar with a circumferential V-notch. E. O. Stitz, USA

3814. Strong, J. G., Resistance-wire strain gauges, *J. roy. aero. Soc.* **54,** 469, 19–26, Jan. 1950.

Paper deals with general theory of resistance-wire gages. Valuable hints are given in connection with suitable equipment and preparation for a strain-gage test. Paper should be of special interest to all not yet acquainted with the method.

C. J. Bernhardt, Norway

3815. Hiltscher, R., A practical lateral-extensometer for determination of the sum of the principal stresses (in German), *Kungl. tekn. Högskol. Handl.* no. 42, 22 pp., 1950.

Analysis of the isochromatic patterns in photoelastic investigations of problems of plane stress yields only the difference, $\sigma_1 - \sigma_2$, of the principal stresses, and, in order to determine σ_1 and σ_2 separately, it is usual to evaluate $(\sigma_1 + \sigma_2)$, either by calculation from the isochromatics, or by a separate experiment. A simple method of determining $(\sigma_1 + \sigma_2)$ is described in this paper. It depends on the fact, pointed out by Mesnager in 1901, that the lateral contraction at a point is proportional to $(\sigma_1 + \sigma_2)$, the coefficient of proportionality involving the thickness of the specimen, Young's modulus, and Poisson's ratio; with phenolformaldehyde, the lateral contraction is small, the maximum and average values in a typical experiment being of order 30 and 10 μ, respectively. A commercial instrument ("Mikrokator"), fitted with a dial gage, was found to possess the necessary sensitivity; one model, for example, having a range of 36 μ and an accuracy of 0.1 μ. Sources of error in the experiment are discussed, and a method of calibration is described. Paper concludes with an example of its use. R. M. Davies, Wales

3816. Durelli, A. J., and DeWolf, T. N., Law of failure of stresscoat, *Proc. Soc. exp. Stress Anal.* **6,** 2, 68–83, 1948.

Results of many tests by authors and others are discussed. Uniaxial compressive stress is found to produce no coating failure, even when transverse extensional strain is five times that which produces failure under uniaxial tensile stress. Isotropic tension, however, is found to produce failure at lower strains than uniaxial tension. Evidence presented clearly establishes limited validity of maximum strain law of failure. However, more adequate con-

trol of test conditions, particularly temperature and humidity, will be needed to provide basis for a definitive law of failure. Reviewer believes that authors' emphasis upon states of stress, rather than strain, needlessly confuses problem. Proposed tentative failure law appears to be theoretically objectionable for this reason. R. E. Newton, USA

3817. Rockey, K. C., Stress analysis using the brittle lacquer process, *Trans. Inst. mar. Engrs.* **63,** 3, 43–47, Mar. 1951.

Paper deals with brittle coating tests conducted with a resin-based lacquer as a substitue for Stresscoat. The techniques used by author for applying the coating are very similar to those used in this country, but the coating has to be heated to 115 C for 24 hours. It develops a sensitivity of 0.0003. Author claims that in dynamic applications the cracks stay open after the load is removed. He also claims that from the intensity of the cracks it is possible to determine quantitative values of the stresses. No examples are given, however, to substantiate these statements. Author has not found a suitable dye to etch the coating. Paper is followed by five pages of interesting discussions.

A. J. Durelli, USA

3818. Fried, B., Some observations on photoelastic materials stressed beyond the elastic limit, *Proc. Soc. exp. Stress Anal.* **8,** 2, 143–148, 1951.

By establishing the stress-optical relationships which are operative beyond the elastic limit of the photoelastic material used, author proposes extending photoelasticity for evaluating stresses in the plastic range. The properties of several materials were studied to determine their suitability for such tests. Cellulose nitrate was judged to be the most satisfactory.

Results are given for a test using a beam subjected to pure flexure. Nadai has shown that the stress distribution may be determined from a bending test by measuring the strain in the outermost fibers in tension and compression corresponding to various values of the bending moment. The stress distribution was determined simultaneously by using Nadai's solution and by photoelasticity. The results obtained by the two methods show excellent agreement. Fringe photographs and a plot showing the effect of creep are included. C. E. Taylor, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 3785, 3789, 3792, 3864)

3819. Tavani, R., On the use of Laplace transformation in the study of bent beams (in Italian), *Atti Ist. Sci. Costr.* no. 11, 8 pp., 1949.

Simple general formulas are derived for the deflection, shear force, and bending-moment variation in beams of either uniform or varying cross section, under either several concentrated loads at arbitrary points along the span or uniformly distributed loads along portions of the span. In each case, a single equation is derived which holds for the entire beam and which is easily applied with the aid of some tabulated functions given in the paper. Applications to continuous beams are discussed briefly.

Bruno A. Boley, USA

3820. Noz, F. J., Contribution to the calculation of the relaxation coefficients of Cross for straight bars with variable moments of inertia (in Dutch), *Ingenieur* **63,** 14, Bt. 25-Bt. 30, Apr. 1951.

Proposal to evaluate integrals, obtained from an application of Maxwell's theorem, by well-known methods of finite differences.

H. F. Michielsen, Holland

3821. Koval'skii, B. S., Elastoplastic deflection of a beam on an elastic foundation (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **77,** 2, 209–211, Mar. 1951.

Author considers effect of linear strain-hardening on elastoplastic bending of beams on elastic foundation. For mild steel beams of rectangular cross sections, the effect of strain-hardening is shown to be appreciable when the elastic zone is reduced to less than the middle third of the beam depth.

General differential equation for elastoplastic bending is obtained by introducing a variable beam stiffness. Neglecting strain-hardening in the case of shallow plastic zones in the beam, above equation is reduced to a nonlinear fourth-order differential equation which may be solved in the form of an infinite power series. Considering linear strain-hardening in the case of deep plastic zones, the general equation may be approximated by a linear fourth-order differential equation very similar to the case of pure elastic bending. B. Bresler, USA

3822. Stevens, O. B., Elementary derivation of the shearing stress distribution, the angle of twist and the warping in a prismatical shaft of elliptical cross section twisted by a torque, *Proc. kon Ned. Akad. Wet.* **54,** 2, 120–129, Mar.-Apr. 1951.

Problem dealt with is of importance for calculation of, say, shaft brackets of elliptical section in torsion and for rough estimate of airfoil sections, such as propeller blades. Derivation is based on pure geometrical comparison between ellipse and major circle and on splitting shearing stress into components parallel to principal axes. It is shown that resultant shearing stress varies linearly along any radius of ellipse and that stresses at points along same radius are all parallel to tangent to ellipse where radius intersects boundary. Plane cross section is distorted to hyperbolic paraboloid. Results agree with Saint-Venant theory which assumes no direct (normal) stress and shearing stresses in two planes only. Georg Vedeler, Norway

3823. Alexander, B., Methods of spiral spring design, *Aircr. Engng.* **23,** 266, 113–116, Apr. 1951.

Assuming that spring can be represented by Archimedean spiral ($r = a\theta$), author derives approximate formula for calculating shift of coils during deflection for case where the outer end of spring is fixed while inner end is attached to arbor. Such a formula will indicate whether adjacent coils will come in contact in operation. In derivation, square of unit advance of spiral is neglected; estimate of error thus involved indicates that this procedure is justified, considering manufacturing variations in practical springs. Application of formulas is illustrated with practical example. A. M. Wahl, USA

3824. Pippard, A. J. S., Stresses in a restrained pipe-line, *J. Instn. civ. Engrs.* no. 3, 170–184, Jan. 1951.

In the construction of pipe lines of large diameter in Iran, temperature variations are so great that proper consideration of the consequent pipe deformations is a major design problem. In practice, the pipe line is supported by anchor blocks and piers which restrain the individual segments from expanding and bending. The resulting restraint stresses may surpass the allowable stress of the pipe material unless the spacing of the supports is properly designed. The load on the anchor blocks must also be known in order that they be adequately constructed.

Author develops equations for the stresses in restrained pipes that are subjected to internal fluid pressure and to temperatures that vary linearly over the pipe cross section. An expression for the anchor block spacing required to prevent failure of the pipe material is obtained by use of the von Mises-Hencky criterion, modified by a suitable factor of safety, i.e., if f_1, f_2, f_3 are the nor-

mal stresses along three mutually perpendicular axes, f_y the yield-stress in direct tension, and N the factor of safety

$$(f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2 = 2(f_y/N)^2.$$

In all his derivations, author uses only elementary engineering theory of stresses and deformations of pipes, which is justified by the small thickness-diameter ratios considered.

Paul Seide, USA

3825. Henriot, G., Tables, nomograms and practical calculations on gearing (in French), Paris, Dunod, 1951, vi + 166 pp. Fr. 1450.

Published for use with "Traité théorique et pratique des engrenages" (Theory and practice of gearing) by the same author, book consists of tables and nomograms which are accompanied by concise explanations and diagrams, making this volume self-contained. Much of the information is already available elsewhere, particularly in handbooks, but some of the tables, especially those on contact ratio, interference limits, and velocity of sliding, should be of interest to gear designers. The nine chapters cover the following subjects: Geometrical parameters of straight and helical gears; contact ratios of straight and helical spur gears; interference of internal and external gears; velocity of sliding; force, torque, and power relations; checking of gears by chordal and pin measurements; and methods for calculating gear trains for given over-all ratios.

The standard tooth systems listed include those used in the United States. The section on gear checking by measurement over pins or balls is limited to equations which require numerical calculations of the required parameters. These might have been reduced to tabular form, as is customary in other references on the subject. G. A. Nothmann, USA

3826. Stamets, W. K., Jr., Dynamic loading of chain drives, *Trans. Amer. Soc. mech. Engrs.* **73,** 5, 655–662, July 1951.

Steady-state dynamic loads in a power chain are computed by superposition of steady chain pull and inertia force due to chain rotation. It is shown that, at low speeds, an increase in the steady inertia force by increasing chain speed may reduce the range of variable stress, and hence increase fatigue life. Optimum chain speed is calculated on this basis and is found to be somewhat higher than usual practice. Experimental work with an optical strain gage and microflash illumination checked the analysis under actual conditions. Effects of transient stresses, such as might be induced by tooth impact, are not included in the analysis or experimental work. Donald E. Hudson, USA

Plates, Disks, Shells, Membranes

(See also Revs. 3797, 3800, 3841)

3827. Karas, K., Plates under lateral impact, *David W. Taylor Mod. Basin Transl.* 135, 20 pp., Apr. 1951.

Translation from *Ing.-Arch.* **10,** 1939.

3828. MacNeal, R. H., The solution of elastic plate problems by electrical analogies, *J. appl. Mech.* **18,** 1, 59–67, Mar. 1951.

Solutions of elastic thin-plate problems for small deflections are formulated in finite-difference form. Solutions of these difference equations are carried out by use of an electric-analog computer. Problems of static deflection, transient vibrations, or normal modes of plates with irregular edges, or any condition of loading or support can be treated with equal facility. Method applies as well to variable thickness and anisotropic plates. Slopes, moments, and shears can be measured directly. Several solutions were obtained on the California Institute of Technology electric-analog computer for the static deflection and normal modes of a rectangular cantilever plate. Comparison of normal mode frequencies for a square cantilever plate as given by the analog computer with direct experimental values indicate the following results: Analog computer gives frequency of lowest mode 8.2% below the experimental value and of second (torsional) mode 17.2% below experimental value. Comparison of analog-computer results with theoretical results indicate further discrepancy. Mode shape comparison shows better agreement. Suggestions for correcting these errors are mentioned. H. G. Lew, USA

3829. Friedman, M. M., Deflection of a circular plate under the action of concentrated loads (in Russian), *Prikl. Mat. Mekh.* **15,** 2, 258–260, Mar.-Apr. 1951.

An equilibrium system of normal concentrated forces is applied to a thin elastic plate of unit radius, some applied over this plate face and some on the periphery. Using a solution of the biharmonic equation (in complex variable form) given by S. G. Lekhnitski in 1938, author obtains the plate deflection as a superposition of deflections due to the individual loads. In each term of this sum there is an (apparently untabulated) definite integral to be evaluated. Robert E. Roberson, USA

3830. Kachanov, L. M., Stability of a plane deflection form beyond the yield point (in Russian), *Dokladí Akad. Nauk SSSR (N.S.)* **77,** 6, 989–992, Apr. 1951.

Author gives relations between stress couples and components of curvature and twist for a bent rod of rectangular cross section, valid beyond the yield point (analogous to Kirchhoff's relations in elastic range); these, with the known Kirchhoff-Clebsch's equations (equations of equilibrium and relations between components of curvature and displacements), form a complete system of 15 equations with 15 unknowns. The case of a rectilinear rod under the von Mises condition of plasticity is discussed, and indications for a more general theory (curved rod, plasticity with work-hardening) are given. For solution of the equations, Ritz's method is proposed. No numerical results are given; nevertheless, author concludes that S. Timoshenko's proposal to calculate with von Kármán's modulus leads to unsatisfactory results.

J. M. Klitchieff, Yugoslavia

3831. Darevskiĭ, V. M., On the question of the action of certain loads on a cylindrical shell (in Russian), *Dokladí Akad. Nauk SSR (N.S.)* **75,** 2, 169–172, Nov. 1950.

Using his previously published work [see AMR **4,** Rev. 1513] author applies it to the problem of a uniformly distributed load along the segments of generator of a radial cylinder and also along the circle of directrix. He gives asymptotical equations for the determination of internal forces for the point placed nearby the vertexes of loaded fields. Witold Wierzbicki, Poland

3832. Clark, R. A., and Reissner, E., Bending of curved tubes, *Advances appl. Mech.* **II,** 93–122, 1951. [New York, Academic Press, Inc.]

Authors repeat the derivation of the differential equations of the general problem of bending of curved thin-wall tubes [AMR 4, Rev. 620]. Problem of bending of a tube with uniform circular cross section is first investigated. The basic parameter is found to be b^2/ah, where b is the cross-sectional radius, a the center-line radius, and h the thickness of the tube. Authors try two methods of solution. Expansion of the solution in trigonometric series leads to an infinite system of linear equations for the coefficients of the series. From this system of equations authors obtain both the results previously obtained by application of the prin-

ciple of minimum potential energy (by von Kármán) and by application of the principle of least work (by Lorenz, Karl, and Beskin). The difference between the two types of results is shown to be a difference between ways of reducing the infinite system of linear equations to a finite system. Using this series method, the larger the value of the parameter b^2/ah the larger is the number of terms which must be taken in the series. This difficulty is overcome by a second method of solution—the asymptotic integration of the differential equation. The accuracy of this procedure has been verified by the overlapping of the ranges of parametric values where both series solutions and asymptotic solutions are valid. In the asymptotic range, simple explicit formulas for the stresses and deformations of the tube are obtained. The method of asymptotic integration is also applied to problems concerning curved tubes of uniform elliptical cross section. Results are also presented in simple explicit form.

T. H. H. Pian, USA

3833. Halbritter, F., The rectangular foundation plate (in German), *Bautechnik* **26,** 6, 181–186, June 1949.

Using a method established by Grein [Pilzdecken, "Theory und Berechnung," Berlin, 1948], author gives approximate formulas and values for bending moments in rectangular one-column foundation plates.

C. J. Bernhardt, Norway

3834. Coan, J. M., Large-deflection theory for plates with small initial curvature loaded in edge compression, *J. appl. Mech.* 18, 2, 143–151, June 1951.

Recently, Samuel Levy gave the exact solution of von Kármán's large-deflection equations for thin, flat, rectangular, isotropic, simply supported plates yielding uniform edge displacements, using a double Fourier series for each of three functions: deflection, stress, and normal pressure [*NACA T.N.* 846, 1942]. Author adapts the above solution to correspond to nonuniform displacements on two stress-free supported edges, and uniform displacements on two loaded edges of plates having a small initial curvature. These boundary conditions are usually those employed in test experiments, making a distinct departure from conventional ones where the boundaries are kept straight and parallel under a distribution of transverse stresses on the edges. The loaded edges may, however, be kept under uniform stresses or uniform strains. Author deduces the numerical case of a square plate whose initial deviation is $^1/_{10}$ of thickness (taking the deflection function limited to first three terms) for sake of comparison with, and as an extension of, the case of the same plate yielding uniform edge displacements by Pai C. Hu, E. E. Lundquist, and S. B. Batdorf [*NACA T.N.* 1124, 1946]. Author's analysis for the determination of critical load leads to results in close conformity with experimental ones.

D. N. Mitra, India

3835. Müller-Magyari, F., Contribution to the buckling of a thin-walled stiffened plate strip under pressure (in German), *Federhofer-Girkmann-Festschrift*, Wien, Franz Deuticke, 275–293, 1950.

Solution of problem of stiffened plates is advanced through development of mathematical procedure for computing single relation to predict both local buckling of plate and buckling of plate-stiffener combination as a Euler panel. Procedure is limited, at present, by expression used for displacement of plate, by assumption of simply supported edges between plate and stiffener, and by assumption, for purposes of simplification, that Poisson's ratio is zero. Basic procedure used is that of strain energy, with displacements expressed by means of a series with undetermined coefficients, and stresses in terms of an Airy stress function. Several simplifying assumptions reduce the complexity of the resulting expressions which, though of interest to analysts, would be tedious in practical application.

Author notes that present paper is only a contribution to overall problem of behavior of stiffened plates and shells in the supracritical range. Further investigations are proposed to evaluate the effects of: (1) Restraint along line of attachment of plate to stiffener; (2) different centroid positions of the effective area; (3) buckling of thin walls of the stiffener section; and (4) using terms in equation for displacement.

Joseph S. Newell, USA

3836. L'Hermite, R., Notes concernng the theory of bent plates and mushroom-floors (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*) no. 192, 26 pp., June 1951.

Pamphlet is obviously intended to interest design engineers in the application of standard analytical methods for solving slab problems. The H. Marcus membrane method and the method of images in connection with the Lagrange plate equation [Timenshenko, S., "Theory of plates and shells," p. 99 and p. 174] are used to determine the bending moments in slabs supported on columns and subjected to various loads. The Southwell relaxation method is also presented. An L-shaped slab is treated in detail.

The paper ends with an interesting method of "cuts and free edges" developed by author to permit one to investigate bending stresses along a free edge of a plate assumed to be derived from a simply supported plate of any shape by cutting to form a free edge. An example is a right-triangular plate supported on the edges adjacent to the right angle and free along the hypotenuse. This triangular plate is derived from a rectangular plate supported on all edges and uniformly loaded. The cut is imagined to be along a diagonal. The central theorem at the base of the method is proved for a simply supported boundary of any shape but enclosing only a simply connected region.

W. H. Hoppman, II, USA

3837. Deverall, L. I., and Thorne, C. J., Some thin-plate problems by the sine transform, *J. appl. Mech.* **18,** 2, 152–156, June 1951.

General expressions for the deflection of thin rectangular plates are obtained for cases in which two opposite edges have arbitrary but given deflections and moments. The sine transform is used as a part of the method of solution, which makes the process of solving the problem much easier. The six general solutions given are those which arise from all possible combinations of physically important edge conditions at the other two edges. Tables useful in application of the method to specific problems are included.

From authors' summary by G. V. R. Rao, India

Buckling Problems

(See also Revs. 3802, 3835)

3838. Weyel, E., Approximation method for the calculation of buckling column elements with stepwise changing moment of inertia (in German), *Bautechnik (Stahlbau)* **28** (**20**), 5, 64–68, May 1951.

Author introduces an effective moment of inertia $I' = I_1\phi_1 + I_2\phi_2 + \ldots$, where $I_1, I_2, \ldots$ are moments of inertia of different parts of column, and $\phi_1, \phi_2, \ldots$ weight coefficients proportional to square of partial lengths $l_1, l_2, \ldots$ of each part with constant moment of inertia and such that $\phi_1 + \phi_2 + \ldots = 1$. Thus, for a partial length equal to zero, corresponding coefficient vanishes, while for $I_1 = I_2 = \ldots = I$ there results $I' = I$. Except for

satisfaction of these two requirements, further definition of ϕ's is arbitrary with the only argument in its favor a good agreement of several calculated examples with results obtained by exact method of Tölke [*Bauingenieur* **9**, p. 601, 1929, abridged in *Stahlbaukalender* 1939, p. 99]. Lack of fundamental justification for choice of ϕ's in case of two different cross sections becomes apparent when author's extension to case of three different cross sections requires complicated correction factors in order to give satisfactory results. Reviewer believes there is no need for empirical formulas when sound approximation methods are available (for Ritz's method applied to one of present cases, see S. Timoshenko, "Elastic stability," 1st ed., p. 131, 1936) and exact method of solution exists. Author's efforts would have been more fruitful in extending Timoshenko's approximate method including, if necessary, additional terms in deflection function; or preparing graphs for practical cases, like ones given in Tölke's basic paper, by solving numerically transcendental equations or determinants involved. George A. Zizicas, USA

3839. Anderson, R. A., Charts giving critical compressive stress of continuous flat sheet divided into parallelogram-shaped panels, *Nat. adv. Comm. Aero. tech. Note* 2392, 31 pp., July 1951.

Compressive buckling of a continuous flat sheet supported by a series of skewed members of infinite bending rigidity and zero torsional rigidity so as to form parallelogram-shaped panels is considered. Increase in buckling strength of such panels over rectangular panels of same area is associated with geometry and restraint imposed by adjacent panels in the formation of a continuous buckle pattern.

Charts of buckling stress coefficients are presented for compressive loads applied parallel and perpendicular to a set of panel sides and interaction curves for various combinations of the two. Solutions are obtained by the energy method, using a simple deflection function. In certain cases, more accurate energy solutions by use of trigonometric series provide spot checks on the accuracy of the simple solution. George Gerard, USA

3840. Eringen, A. C., Buckling of a sandwich cylinder under uniform axial compressive load, *J. appl. Mech.* **18**, 2, 195–202, June 1951.

A thick-walled hollow cylinder under the following boundary conditions is treated as a first problem: Displacements at the lateral surfaces are arbitrary functions of the longitudinal coordinates, and at both ends the radial displacements and the longitudinal surface forces vanish. The stress distribution of this problem is used by author for the state of stress in the intermediate layer of a sandwich cylinder. The stability problem of the latter can be solved in a simple manner if the boundary conditions at the contact surfaces between the core and the outer skins are satisfied only approximately. In this way, author obtains two differential equations which can be solved by Bessel functions. Results are applied to two types of materials of the outer skin and to a special material for the core. As for the stability problem of the sandwich plate, which is contained in these equations as a special case, author seems to agree with previous research (in regard to the most important results).
H. Neuber, Germany

3841. Bunich, L. M., On a functional for the solution of the stability problem of thin shells (in Russian), *Prikl. Mat. Mekh.* **14**, 6, 670–671, Nov.-Dec. 1950.

The formula of H. M. Mushtari [*Izv. Fiz.-mat. obschch. Nauchno-issl. inst. mat. mekh. Kazan. Univ.* **11**, 3, 1938] for a functional Q, containing the second-order terms of the work of stresses during the transition of a shell from one state of equilibrium to another, does not take into account the variation of external forces during the deformation. Applied to the problem of elastic stability of a short cylindrical shell under uniform external pressure, this functional gives results which do not agree with the exact solution by M. Levy. Author proposes, therefore, an additional term to the functional so that it gives correct results in the above-mentioned problem. Then he applies the new functional to determine the critical pressure on two short semicylindrical shells joined together along both opposite generators by pin joints, and obtains for the critical pressure the formula: $P = 0.8\,EJ/R^3$ (E Young's modulus, J moment of inertia of section, R radius), which is in complete agreement with a former result by V. F. Segal. Anton Kuhelj, Yugoslavia

3842. Sievers, H., Instability of a strut with elastically rotating supports (in German), *Bauingenieur* **26**, 2, 38–41, Feb. 1951.

Author treats case mentioned in title in connection with equivalent problem of strut with elastically deflecting supports. Formulas for latter (flexural) cases are derived extremely briefly. Author then impugns conclusion about limiting case as given by Bleich in his early work, suggesting a new stability condition in addition to the classical one. Reviewer cannot agree with author's motivation of said suggestion and thinks Bleich's conclusion, as far as author's Eq. 14 is concerned, is correct.

Solutions for torsional case are presented for a simple strut with free-warping ends and natural axis of rotation, and for a continuously supported strut with elastic rotation about a predetermined axis. Case of spaced supports is virtually not dealt with. Reviewer considers application of Eq. 18 (in case b) of presented example to be erroneous.

In resulting Eq. 12, radical signs should be added to terms π/β. In Eq. 18, summation under radical sign should be substraction.
H. F. Michielsen, Holland

3843. Lombardi, G., Built-up wooden strut (in German), *Schweiz. Bauztg.* **69**, 22, 301–303, June 1951.

Stress analysis of built-up wooden struts is discussed. The strut is made up of various sections and is loaded by concentric axial compressive loads. The connection between the sections is assumed to be continuous and elastic. The differential equations of buckling of the built-up strut are set up and solved for the cases where the connection between the various parts of the strut is rigid, nonexistent, and somewhere between rigid and nonexistent (the "general case"). Wooden struts containing five, four, three, and two parts are discussed. Paul Torda, USA

3844. Ericksen, W. S., and March, H. W., Compressive buckling of sandwich panels having facings of unequal thickness, *For. Prod. Lab. Rep.* 1583-B, 30 pp., Nov. 1950.

The buckling loads of rectangular sandwich panels with facings of unequal thickness are calculated by using the energy method. The facing and core materials are taken to be orthotropic, but it is assumed that the materials in the two facings are the same and similarly oriented. Four boundary conditions are considered: (1) All sides simply supported; (2) loaded edges simply supported and the remaining edges clamped; (3) loaded edges clamped and the remaining edges simply supported; and (4) all edges clamped. C. T. Wang, USA

3845. Dutheil, J., Study on buckling of tubes (in French), *Ossature métallique* **16**, 2, 87–95, Feb. 1951.

Author gives an approximate method for calculating the failing load of thin-walled tubes. When the thickness/radius ratio exceeds a certain value depending on the material, failure is by

plastic yield in the manner of a solid strut. For thickness/radius values below this value, failure is precipitated by local buckling analogous to the buckling of a strut on an elastic foundation. By adjusting the constant of the foundation to give the value for elastic buckling established by Donnell and others, author is able to consider plastic buckling of the cylinder as equivalent to the much simpler problem of plastic buckling of a strut on an elastic foundation. Further, having deduced the critical stress for local plastic buckling of the tube, author uses this stress as equivalent to the yield stress when dealing with long struts in which Euler-type failure is predominant.

Also given is a criticism of methods commonly used at present in France for calculating struts. P. C. Dunne, England

Joints and Joining Methods

(See also Revs. 3855, 3884)

3846. Weck, R., Transverse contractions and reaction stresses in butt welded mild steel plates, *Admiralty Ship Weld. Counc., Lond., Rep.* no. 4, 46 pp., 54 diagrams, 1950.

Incentive for study was failure of some welded ships during World War II. Three welding procedures were used: Conventional low current, deep penetration, and medium penetration. Specimens 1/4, 3/8, 1/2, and 3/4-in. thick were manually welded with no restraint, with moderate restraint, and with rigid restraint. Considerable scatter resulted for identical experiments because of variation of speed in manual welding.

Welding procedures using small electrodes and low currents caused greater contractions than the deep-penetration procedures. Stresses were much greater near the weld and greater at the top surface. Average reaction stresses were about 5000 psi, while stresses as high as 45,000 psi were found on the top surface.

Fine cracks were found in the first run of some specimens where low-current procedure was used. Reviewer feels that this is especially significant where repeated loadings are applied to the structure.

Total contraction is due to reaction stresses, residual stresses, and plastic flow.

Report also contains a review of literature and mathematical criterion for states of initial stress. Glen N. Cox, USA

3847. Flanigan, A. E., Kaufman, M., and Emery, E. M., Strain-aging of arc welds in mild steel, *Weld. Res. Suppl.* **15,** 3, 146s–160s, Mar. 1950.

This investigation deals with the strain-aging characteristics of AWS E6010, E6012, E6016, and E6020 welds in mild steel. Specimens were of the longitudinal bead-on-plate type notched transverse to the weld. The notch was cut to a depth of 0.020 in. below the plate surface; weld penetration was approximately 0.110 in. In general, the procedure was as follows: (1) A 9 × 4 × 3/4-in. specimen was welded and notched; (2) prestraining was introduced at room temperature by bending to a small angle; (3) specimen then was aged at 70 F or at 212 F; and (4) bending was continued at −40 F to failure. Performance criteria were bend angle (1) at the first indication of a crack, (2) at complete fracture of the weld, and (3) at complete fracture of the specimen. Specimens were tested at −40 F to render material outside the weld unable to resist propagation of cracks originating in the weld; thus, the study dealt with performance of weld deposit and was largely independent of base metal used.

The following factors were investigated as to their effect on strain-aging embrittlement: (1) degree of prestraining; (2) duration of storage at 70 F before and after prestraining; and (3) duration of storage at 212 F after prestraining. To determine the effect of prestrain and aging on subsequent capacity for bending, degree of prestraining was plotted against additional deformation necessary to fracture. The effect of temperature and duration of storage was evaluated by plotting bend angle at fracture vs. storage time for prestrained and nonprestrained specimens.

The following observations were made: (1) An aging effect occurred in all welds tested, including a low hydrogen E6016 deposit; (2) storage at room temperature following prestraining resulted in an embrittlement that reached a peak within a week; additional storage of about a month sufficed to effect full restoration of ductility; (3) an increasing period of storage between welding and prestraining caused a decrease in the severity of subsequent strain aging; (4) aging at 212 F greatly accelerated attainment of the peak embrittlement and subsequent restoration of ductility; and (5) the aging effect was believed to reside in the weld metal itself rather than in the weld heat-affected zone, or in the parent plate.

Reviewer found it necessary to consider the possibility of quench aging although authors used the term "strain aging" exclusively. For example, the effect of storage in producing initial embrittlement and subsequent amelioration appears to be contradictory to the mechanics of strain aging as revealed by hardness studies (in the case of strain aging a maximum in hardness is produced which does not diminish after as long as 1000 hr at temperatures as high as 400 F; whereas, in quench aging, complete overaging is developed at 212 F in a matter of a few hours). Likewise, the fact that welded specimens (1) aged in excess of 2 hr at 212 F resisted propagation of the weld crack at −40 F and (2) aged less than 2 hr at 212 F offered no resistance to the weld crack at −40 F, indicates that quench aging is operative in the weld heat-affected zone. Carl E. Hartblower, USA

Structures

(See also Revs. 3786, 3828, 3842, 3843, 3891)

3848. Bignoli, A. J., and Basaldúa, J. J., Errors in calculation of statically indeterminate structures (in Spanish), *Cienc. y Técn.* **115,** 581, 314–326, Nov. 1950.

The concept of sensibility of a magnitude to given variations of another is analyzed in order to determine the sensibility of the final results to the errors which arise from the various hypotheses and numerical computations necessary in the solution of a given problem. Method is then applied to the solution of a statically indeterminate structure to show the influence on the accuracy of the results of an adequate solution of the redundants.

Finally, practical criteria are given for the selection of the unknowns so as to reduce the sensibility of the system and hence minimize the errors. Ernesto Saleme, USA

3849. Klöppel, K., Theory of compound steel structures in statically indeterminate systems with consideration of creep effect (in German), *Bautechnik (Stahlbau)* **28 (20),** 2, 17–23, Feb. 1951.

Beginning with the general elastic equations for an n-fold indeterminate structure, assuming reinforced-concrete construction, and considering temperature effects, shrinkage and creep, and settling of the supports, expressions are developed for bending moments and displacements. Particular attention is paid to creep in concrete, which is known to have viscoelastic properties. The creep law is taken to be $\phi_t = \phi_e(1 - e^{-t})$, in which ϕ_t is creep rate, ϕ_e is final creep rate, and t is time. A numerical example is included. Simplification of the results for practical application is to be the subject of further work.

A. D. Topping, USA

3850. Lamberg, R., Analysis of closed frames by distributing angular deformations (in German), *Bautechnik (Stahlbau)*, **28 (20)**, 4, 41–43, Apr. 1951.

Author points to C. V. Klouček's method analyzing frame structures by distribution of deformation [see Dr. Klouček's books: "Das Prinzip der fortgeleiteten Verformung als Weg zur Ausschaltung der Unbekannten aus dem Formaenderungsverfahren," Berlin, 1941; "Rozvod deformace," Prague, 1940, and AMR 2, Rev. 859] and presents a new method for exact solution of closed frame structures by means of determinants. The solution on the basis of Gauss' algorithmus is relatively simple if the structure consists of one or two closed frames; it becomes, however, very complicated for multiple-row and story frames. The intricacy is especially caused by the necessity of determining angular deformations of the joints for every type of loading, so that the advantage of both Klouček's and Lamberg's methods becomes seriously questionable. J. J. Polivka, USA

3851. Efsen, A., Calculation of the general Vierendeel frame (in Spanish), *Cienc. y Técn.* **116**, 583, 1–26, Jan. 1951.

This solution to the general case of a plane Vierendeel frame is based on the method of primary moments, with which the reader is assumed to be familiar. Axial deformation of the members is neglected and certain combinations of the external supporting forces are assumed. A solution for the general case of the plane quadrilateral element is given and utilized in a series of combinations representing special cases. Only simple algebraic equations are involved in the solutions. Glenn Murphy, USA

3852. Dall'Aglio, B., Simply connected plane frames calculated without equations (in Italian), *G. Gen. civ.* **88**, 2, 4; 93–102, 225–231; Feb., April 1950.

A rigid-jointed plane frame, loaded in one field transversely to one of its members, is considered here. Effect of the unloaded bars is taken into account by imaginary members, whose effective length and bending rigidity can be obtained from formulas presented in paper. Determination of the bending moments in the frame is thus reduced to a purely numerical (without equations) tabular computation. Extension of the method to frames including axially loaded bars (neglecting beam-column effects) is easily accomplished. Part I takes up frames with rigidly fixed supports; Part II, frames with hinged supports. Numerical examples are worked out in detail. Bruno A. Boley, USA

3853. Rambøll, B. J., Calculation of frame systems taking into account the bending of the columns (in German), *Publ. int. Assn. Bridge Struct. Engng.* **9**, 429–452, Nov. 1949.

In many light continuous structures being built today, the lateral deformations of the columns have an appreciable effect on the bending moments existing throughout the structure. Author presents a method, based upon work by Efsen and Nøkkentved, for taking these deformations into consideration in the structural analysis. The method involves first the evaluation of the "frame constant" for each member and joint of the structure. The end moments are then obtained by substituting in the appropriate expressions.

Author points out that actual failure load must be computed when investigating the capacity of structures in which lateral column deformations are important. This is because of the lack of even an approximate proportionality relation between loading and moments. But in the analysis for failure loads, the stresses in some members will usually be found to exceed the proportional limit. Author states that this situation can be dealt with by using the value of the modulus of elasticity corresponding to the stress at failure load, when computing the frame constants. (Since the failure load is not known in advance, this procedure would, of course, involve successive approximations.) However, he apparently ignores the fact that in most members stressed beyond the proportional limit, E will vary throughout the member. In such cases, the expressions which he develops are no longer valid.

Howard Simpson, USA

3854. Ferguson, P. M., and White, A. H., The statics ratio for analysis of frames that deflect, *Univ. Texas Publ.* **5022**, 43 pp., Nov. 1950.

Authors' technique facilitates analysis of frames with three or more degrees of freedom (referring to relative displacements of panel points). It is an extended and generalized version of a method suggested by Grinter [*Trans. Amer. Soc. civ. Engrs.* **99**, p. 610, 1934] and has advantages over previously available methods which involved successive corrections or solution of simultaneous equations. It is convenient to use concept of a "model frame" identical with one under analysis but dissociated from given loading and then subjected to arbitrary system of panel-point displacements. "Statics ratios" can be of various kinds; for a multi-story single-span rectangular frame a natural statics ratio is the ratio of the sum of the shears in the members at any level of the model to the actual total shear carried at the same level by the loaded frame. As many statics ratios exist as degrees of freedom. If the deflections, moments, etc., of the model are in constant proportion to those of the loaded frame, then all statics ratios are equal. Method uses moment distribution, starting with fixed-end moments corresponding to a plausible deflection system. After a few cycles of moment distribution, statics ratios are computed. Lack of equality indicates need for introducing corrective fixed-end moments, in amounts which can be roughly estimated from the desired changes in statics ratios. Corrective moments may be pictured in terms of appropriate deflection patterns, which are discussed for various types of complex frames. Accuracy of method is discussed in terms of examples. Method appears speedy, simple, and flexible.

P. S. Symonds, USA

3855. Joukoff, A. S., Experimental research on spot-welded structures (in French), *Rev. Soud.* **6**, 3, 136–143, 1950.

Description of tests on statically and dynamically loaded spot-welded roof trusses of light-gage steel. Failure under static load was characteristically buckling of upper chord, whereas dynamic load produced failures at the spot-welded joints in the material surrounding the fused areas, but not in the spot welds themselves. Fairly obvious conclusions call chiefly for new methods of treatment of spots after welding. Paper's importance is largely in pointing out possibilities of spot-welding in light-gage-steel house construction. Gordon P. Fisher, USA

3856. Nowiński, J., Stresses and deformations of a two-spar cantilever wing, *Techn. Lotn.* **6**, 1, 2–13, Mar. 1951.

Author presents exact analysis and stress formulas of a two-spar wing having any shape of trapezoidal cross section, constant throughout its length. It is assumed that the skin of the wing takes all shear whereas direct stresses are transmitted to the corner struts of any shape; the wing is subjected to a transverse load acting at free end. Also, the effect of stiff restraint is considered. It is shown that the axis of rotation is not, in general, a simple line, but depends on the loading; nor, generally, does it pass through the shear centroid of the loaded section. It is also proved that if the concentrated load passes through the shear centroid of the end section, all other sections of the wing (except the end section) are subjected to rotation. Author refers to previous papers on the same subject by Grzodzielski and Billewicz, Nale-

-kiewicz, Hadji-Argyris and Dunne, and to fundamental investigation by M. T. Huber. J. J. Polivka, USA

3857. Troche, A., On the calculation of wooden post trusses (in German), *Bauingenieur* **26**, 1, 18–23, Jan. 1951.

In trapezoidal wooden post trusses, the length variations of the various bars arise from different causes, which are: (1) Elastic stresses in these bars; (2) shrinkage; and (3) displacements due to incomplete contact between the pieces and local deformations in the joints. The two last causes are much more important than the first.

Author gives tables of formulas which permit these length variations to be taken into account in calculating the statically indeterminate structure. In a numerical example, he shows the relative correction due to this effect to be of the order of 20%.

Ch. Massonnet, Belgium

3858. Jaeger, C., Modern trends in arch dam construction and design, *Civ. Engng. Lond.* **45**, 526, 527, 528; 230–233, 304–306, 379–387; Apr., May, June 1950.

A comprehensive review of recent work on research, design, and construction of arch dams. In construction, dams built in the United States feature heavy sections, whereas in Europe, high, thin dams (typified by the Italian Cupola dams) appear to be the trend. In design, "trial-load method" has been generally adopted in the United States. A dam is assumed to consist of a grid of horizontal arches and vertical cantilevers. Stresses and deformation are then determined by successive approximation. In Europe, more straightforward procedures have been preferred.

Author also emphasizes modern trend of paying utmost attention to boundary conditions in analysis and the lack of knowledge on fundamental aspects of problem.

Ai-Ting Yu, USA

3859. Bodger, W. K., A method of inferring the strength of structures at high temperature from room-temperature model tests, *Proc. Soc. exp. Stress Anal.* **8**, 2, 63–68, 1951.

By using a model material with a stress-strain curve (at room temperature) geometrically similar to the stress-strain rate curve of a prototype material (at high temperature for fixed time), author determines a factor which relates the load-carrying capacity of the prototype to that of the model. This factor is applied to a load deformation curve of the model to give the load carried by a geometrically similar prototype. Author applies method by determining the load-carrying capacity factor for the prototype material S-590 with operating temperature of 1350 F and design life of 100,000 hours and the model material 248 aluminum. Applying the calculated factor to a model load-deformation test curve, author states that the solution for a turbine problem using the prototype material was entirely satisfactory. B. E. Gatewood, USA

3860. Thompson, E. K., Concrete shell structures in the West *Architect. Record* **32**, 2, p. 3, Feb. 1951.

Short description and pictures of the first large reinforced-concrete dome of thin section in the United States, constructed in 1909 over the Second Church of Christ Scientist in Los Angeles (A. F. Rosenheim, architect and A. C. Martin, engineer), and of several recently built concrete shell structures in the West (shell dome of San Francisco planetarium; barrel shell roof, 253 ft in span, of Municipal Stadium in Denver, Colo.; multiple barrel shell of Western Electric Co., Denver, Colo.; and others), pointing to structural principles and advantages, especially to great economy. Thicknesses of concrete shells down to $\frac{3}{4}$ and $1\frac{1}{4}$ in. are not unusual. J. J. Polivka, USA

3861. Linton, W. P., Nomographic analysis of rectangular sections of reinforced concrete, *Trans. Amer. Soc. civ. Engrs.* **115**, 107–120, 1950.

Author gives three nomograms for design or analysis of reinforced-concrete members subject to flexure and normal loads. Theory is usual theory of reinforced-concrete design. Singly and doubly reinforced beams subject to moment only may also be treated as special case when normal load is zero. Appendix explains concisely the construction of nomograms. Reviewer believes method is an improvement on the usual procedure of employing graphs and tables. Frank A. Blakey, Australia

3862. Evans, R. H., Research and developments in prestressing, *J. Instn. civ. Engrs.* **35**, 4, 231–261, Feb. 1951.

Paper is in part a review of previously published research work, with comments by author. Topics discussed are bond stress distribution in pretensioned prestressed concrete, heat-treated concrete, composite and partially prestressed concrete, ultimate strength of prestressed concrete beams, and new developments in prestressing.

The paper by G. Marshall [*Mag. Conc. Res.* no. 3, p. 123, Dec. 1949] is repeated practically in toto. Author adds calculations to show that the controversial "wedge effect" does little to increase anchorage strength. He attributes the greater bond strength of prestressing reinforcement as compared with reinforcement in ordinary reinforced-concrete beams to (a) increased frictional resistance owing to the lateral expansion of the wires, (b) tendency in the latter case for the tensile concrete to pull away from the reinforcement, due to Poisson's ratio, (c) greater slip movement near ends of pretensioned beams, enabling Jenkin's principle of dilatancy to take effect.

High-temperature curing is often used to obtain high early strength, enabling early removal of forms and tensioning devices. Author finds that for 1 to 4 hours of steam curing, the optimum pressure is 30 psi. Effect of steam curing on strength and shrinkage is discussed, with the presentation of some new data.

For computing the ultimate strength of prestressed beams, author recommends a modification of Whitney's theory, as described by the author [*J. Instn. civ. Engrs.* **21**, p. 98, Dec. 1943]. Some new supporting data are presented. The paper closes with a discussion of new developments in prestressing materials and applications. Howard Simpson, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 3757, 3797, 3821, 3849, 3878, 3883, 3893, 3894, 3896, 4020)

3863. Stüssi, F., The foundations of the mathematical theory of plasticity and the experiment (in German), *Rev. Acad. Cienc. Madrid* **44**, 123–138, 1950.

The main portion of the paper is concerned with the results of tests on thin-walled aluminum-alloy tubes. By the application of an interior pressure and an axial compression, a state of pure hoop tension in the plastic range is produced. On this, further stresses are superimposed by changing the interior pressure and axial compression and applying a torque. The axial and circumferential extensions and the angle of twist caused by these additional stresses are measured. Under pure hoop tension, the plastic compressive strain in the axial direction is found to be less than half the plastic tensile strain in the circumferential direction. Author concludes from this that the plastic deformation is not volume-preserving. Actually, a more likely explanation would seem to be that the material is not isotropic. This seems to be confirmed by some of the subsequent tests. Author claims that his experiments prove the basic assumptions of the mathematical

theory of plasticity to be in error. To reviewer, this claim appears to be far too sweeping because the only theories with which the author compares his results are of the "deformation" or "finite" type. It is well established that theories of this type agree with experiments only if the principal axes of stress remain fixed with respect to the material and if the ratios between the principal components of the stress deviation are preserved during the test. Since the author's tests are not conducted in this manner, the discrepancy between the experimental results and the predictions furnished by the finite theories used by the author is not surprising. It would be interesting to confront the author's experimental results with a "flow" or "incremental" theory which takes account of the initial anisotropy of the material.

W. Prager, USA

3864. Horne, M. R., The plastic theory of bending of mild steel beams with particular reference to the effect of shear forces, *Proc. roy. Soc. Lond. Ser. A*, **207,** 1089, 216–228, June 1951.

Paper calculates the stress distribution in rectangular and I-section cantilevers under tip load, for the case of a material with a marked yield point. Attention is directed to regions of plasticity developing at the extreme fibers due to direct stress and at the center due to shear stress. Distribution of stress in these regions is investigated with unusual mathematical rigor. Calculated slip planes are confirmed by experiments quoted.

W. S. Hemp, England

3865. Edelman, F., and Drucker, D. C., Some extensions of elementary plasticity theory, *J. Franklin Inst.* **251,** 6, 581–605, June 1951.

Paper contains a valuable survey of yield or loading criteria for work-hardening materials from which incremental strain theories may be derived. Most theories at present in use fail to some extent to agree with experiment, owing to a lack of flexibility suitable to the conditions of the problem. The criteria considered here are designed to give the required flexibility. Eight types are considered, five of which allow for a Bauschinger effect. Among them are criteria dependent on stress alone and on both stress and strain. In the latter cases the strains appear only to the first order. Four of the criteria involve both the second and third stress invariants. Paper also contains a summary of the application of the tensor notation to the specification of stress and strain and an introduction to incremental strain theory which does not appear to add to the value of the paper. Paper will be very helpful to those who are trying to obtain the maximum amount of information from experimental data.

W. M. Shepherd, England

3866. Rotherham, L. A., Creep of metals, London, Institute of Physics, May 1951, 80 pp. 15s.

This short monograph, published under the auspices of the Institute of Physics as one of a *Physics in Industry* series, concerns itself primarily with the physics of creep phenomena in metals. Since author is a metallurgist, treatment deals with the problem more from the viewpoint of the metallurgist and physicist than from that of the engineer or designer. However, book contains considerable fundamental data and information useful in engineering work.

First chapter discusses fundamental characteristics of the technical creep curve including effects of stress, temperature, creep recovery, and grain size. Next two chapters discuss slip process, crystalline flow, and metallographic features of creep. Additional sections deal with grain-boundary creep, transient creep (including various theories), steady-state creep including chemical rate and dislocation theories, and tertiary creep. A short section on so-called stress-rupture tests is also included. Because of the very brief treatment of many of these subjects, reader is advised, where possible, to consult selected list of references given. Final chapter treats development of creep-resistant alloys; author suggests several lines along which further work might profitably proceed.

No discussion is given of creep under biaxial or triaxial stress, nor is the subject of relaxation covered.

A. M. Wahl, USA

3867. Ferry, J. D., Fitzgerald, E. R., Johnson, M. F., and Grandine, L. D., Jr., Mechanical properties of substances of high molecular weight. X. The relaxation distribution function in polyisobutylene and its solutions, *J. appl. Phys.* **22,** 6, 717–722, June 1951.

Earlier measurements [Marvin, Fitzgerald, and Ferry, title source, **21,** 197–203, 1950] are extended in present work to cover wider temperature range and more dilute solutions. Dynamic rigidity and viscosity have been measured with twin transducer for solid and solutions in xylene and Decalin. Results are now available for times from 10^{-5} sec to 10^{6} sec. Dynamic rigidity shows plateau from 0.1 to 10^{4} sec rises sharply for shorter times and drops sharply for longer times. Results for solutions give same shape as that for solid.

H. Kolsky, England

3868. Lethersich, W., The rheological properties of dielectric polymers, *Brit. J. appl. Phys.* **1,** 11, 294–301, Nov. 1950.

Dielectric polymers are used as construction materials as well as insulators; therefore, their elastic properties under long time load are of interest. Author reports that tests made for periods up to 18 months on dielectric polymers indicate that, under static load, polymers continue to elongate after loading. Although the rate of creep diminishes with time, there is no indication that it approaches a terminating value. When load is removed, the recovery curve follows extension curve for several hours, then recovery proceeds at a lower rate. For the polymer perspex at the end of two-years' recovery there is no indication that rate of recovery approaches a terminating value. Dynamic tests show that dynamic modulus G increases with frequency, since less creep occurs at high frequency. However, coefficient of internal friction decreases with frequency. Creep is wholly elastic and may be represented by a series of Kelvin elements in series, each having relaxation time τ.

James F. Goodwin, USA

3869. Charron, F., Study of plastic bodies under high pressure (in French), *Publ. sci. Min. Air, France*, no. 247, 30 pp., 1951.

The growing need for basic information dealing with the flow of highly viscous substances makes this a valuable contribution. Unfortunately, the experimental portion is rather brief, although the description of a device for measuring the viscosity of greases under pressure is most welcome. Rather curiously, the effect of viscosity or greases is more marked than is usually supposed. No attempt is made to correlate the experimental study with the general theory of non-Newtonian flow introduced in the first portion of the work. The author concludes: (a) Plastic substances exhibit imperfectly the existence of a friction term and a viscosity term independent of the motion of the substance; (b) the friction term varies only slightly with pressure. In the experiments included in the investigation, friction varied almost linearly with pressure; (c) the viscosity term increases exponentially with pressure, but varies considerably with different greases.

J. M. DalleValle, USA

3870. Rigden, P. J., Rheology of suspensions of high solid concentration, *Nature* **167,** 4240, 197–198, Feb. 1951.

Suspensions of mineral powders in a bitumen were examined for a correlation between log relative viscosity at a single shear stress and the ratio of solids concentration to mean pore radius. The relationship is roughly linear for a group of systems comprising sixteen powders of seven mineral types.

John W. Cook, USA

3871. Reiner, M., Rigden, P. J., and Thrower, E. N., The volume flow of asphalt, *J. Soc. chem. Indust.* **69,** 257–260, Sept. 1950.

Change in volume and elongation of samples of asphalt during constant-load creep tests have been measured. Deformation due to shear has been distinguished from that due to change in volume. Both shear and volume flow have a permanent and a recoverable part. The behavior is analyzed in terms of a model containing elastic springs and viscous dashpots. A small discrepancy is noted in the recovery curves predicted by the model. This is due to a peculiar strain-hardening or "backlash" of the asphalt.

Evan A. Davis, USA

3872. Scheele, W., and Timm, T., On the question of treatment of steady deformation processes in polymerisates (in German), *Kolloid Z.* **120,** 1–3, 103–111, Jan. 1951.

Flow curves on non-Newtonian polymers obey power law $v = k\tau^n$, v flow velocity, τ shearing stress, k and n constants. For purposes of analysis, $W(x)$, called deformation work (Verformungsleistung), is defined as ${}_0\int^x \tau\, dv = k^{-1/n} \cdot n/(n+1) \cdot x^{(n+1)/n}$. Published data by Scheele and co-workers, obtained with Höppler consistometer, obey $W = A \exp(Q/RT)$, T temperature, R gas constant, A, Q constants, Q being activation energy. Survey of data on many polymers leads to conclusions: High polymers show Newtonian behavior ($n = 1$) at very low τ or at high T. At moderate τ and T, they show non-Newtonian behavior ($n > 1$). Under latter conditions $Q = a - b \ln [\eta]$. Q increases with intermolecular forces, is decreased by plasticizers, increases at lower x. Exponent n increases with degree of polymerization.

Reviewer deplores use of the inadequate experimental and analytical methods in this work. Capillary or rotating cylinder viscometer would give absolute viscosities; and analysis of data by theory of reaction rates, developed by Eyring and associates, would lead to a more complete understanding of viscous flow of materials tested.

Melvin Mooney, USA

3873. Wobser, R., Rheological measuring methods in industrial practice (in German), *Instrum. Measur. Conf., Stockholm, Trans.* **165–176, 1949.**

Three consistometers designed by Höppler are described, of which one (viscometer) is well known. The others permit variation of stress. Excessive claims are made for them by author. Consistency curves for ice from measurements made with such instruments are shown, suggesting that ice has a yield value, which is questionable.

M. Reiner, Israel

3874. Sylwestrowicz, W., and Hall, E. O., The deformation and ageing of mild steel, *Proc. phys. Soc. Lond., Sec. B,* **64,** part B, 378B, 495–502, June 1951.

Studies were made of the propagation of stress and the development of Lüder's bands in small wires ($\frac{1}{2}$ to 1 mm in diameter) of three different steels. Grain size of each material was varied over a wide range by annealing in a vacuum. As the grain size was reduced, both the upper and lower yield points were found to increase markedly and a greater difference was observed between the upper and lower yield points. After a small plastic strain, aging at room temperature or at slightly elevated temperatures restored the upper yield point to a level which depended on both the temperature and time of aging. After more severe aging, it was found that subsequent plastic deformation occurred by new Lüder's bands spreading through the old strain-aged bands at a much higher stress level than that required for the original plastic deformation. The stress for propagation of the secondary band may be as much as 40% above that required for the primary slip band. Authors discuss several theories that have been proposed to explain the occurrence of an upper yield point and suggest that the results of tests by others on single crystals indicate that the grain boundaries exert an important influence on the phenomenon. Their own work gives no indication of the nature of the grain-boundary reactions but shows the important influence of the number of grain boundaries on the mechanical properties of mild steel.

T. J. Dolan, USA

3875. Eshelby, J. D., Frank, F. C., and Nabarro, F. R. N., The equilibrium of linear arrays of dislocations, *Phil. Mag.* (7), **42,** 327, 351–364, Apr. 1951.

A method is given for determining the equilibrium positions of a set of like dislocations in a common slip plane under the influence of a given applied stress. The equilibrium positions are the roots of certain polynomials. According to an indication by H. Heilbronn, authors base their analysis upon the connection between a set of algebraic equations and a differential equation, the solutions of which are orthogonal polynomials. The case of several locked dislocations is discussed in detail and the resulting stress distribution is compared with that produced by a crack with freely slipping surfaces.

F. Schultz-Grunow, Germany

3876. McAdam, D. J., Jr., The stress-strain energy relationship for metals, *Trans. Amer. Soc. Metals* **43,** 970–992, 1951.

From curves derived from previous tensile tests on copper, curves of constant strain-hardening are derived, in which the criterion of strain-hardening is the yield strength at a given temperature. It is found that the external work necessary to produce a given amount of strain-hardening is several times greater when the strain is introduced at high temperature than when a low temperature is used, particularly when the given amount of strain-hardening is large. Author points out that this behavior is incompatible with the results of Taylor and Quinney if one assumes that the latent (or stored) energy is the same in two specimens when the amount of strain-hardening is the same. Taylor and Quinney found that the ratio of stored energy to work done on the specimen was independent of strain up to a considerable strain, so that the above assumption combined with Taylor and Quinney's result would lead to the conclusion that the work done should be the same when the strain-hardening is the same.

D. J. Lubahn, USA

Failure, Mechanics of Solid State

3877. Tipper, C. F. E., The fracture of mild steel plate, *Admiralty Ship Weld. Comm. Rep.* no. 3, 42 pp., 1948.

Report of experimental results of research by Engineering Laboratory, Cambridge. Presentation covers a description of: Fractures in actual ship plate; experiments designed to reproduce similar fractures in laboratory; development of a notch test; metallurgical investigations; effect of notches on ductility; and fracture of mild steel. The data are reported in extended tables.

Results show a large number of plates sent for test are liable to cleavage fractures at normal working temperatures even though

they conform to specification as regards composition and tensile test. When transition temperatures are below 0 C, chances of service failure are reduced. When the ductility was such that stress relief by plastic flow can be achieved before the formation of a crack, failure is less likely to occur. Stress relief by plastic flow is less in large structures. C. O. Dohrenwend, USA

3878. Rosenthal, D., and Mitchell, W. D., The influence of biaxiality on notch brittleness, *Weld. Res. Suppl.* **15,** 9, 409s–421s, Sept. 1950.

Experimental data are presented which show the combined effect of increase in thickness of specimen and depth of notch on the notch sensitivity of mild-steel flat bars subjected to tensile and impact loads. The tests were conducted on bars with different notch radii and depths, but the stress concentration at the root of the notch was kept the same in all cases.

Experiments indicated clearly that the transition temperature for thin bars was sensibly the same for deep, medium, or shallow notches. As the thickness of the bars was increased the transition temperature was raised, being highest for bars with the deep notches. This embrittlement is ascribed to the effect of the biaxial stresses in the direction of the bar thickness and in a plane normal to the thickness. These biaxialities produce a state of triaxial stress behind the notch which enhances the embrittlement of the material and possibly locates the region from which failure is initiated. The appearance of certain failures of notch bars seems to support the location of the triaxial state of stress. However, further investigations of this phase of embrittlement are required. Elio D'Appolonia, USA

3879. Fenner, A. J., Owen, N. B., and Phillips, C. E., The fatigue crack as a stress-raiser, *Engineering* **17,** 1, 4452, 637–638, May 1951.

Authors find that fatigue cracks are formed after relatively few stress cycles (5×10^3) and at low nominal stresses (1.7 tons per sq in.). Magnetic crack detector and metallurgical sectioning are used to establish the presence of cracks. Cracks are found to propagate very slowly so that authors postulate that the S/N curve represents the time taken for cracks to develop across the section rather than the time required for formation of a crack.

William J. Anderson, USA

3880. Weisman, M. H., and Kaplan, M. H., The fatigue strength of steel through the range from $^1/_2$ to 30,000 cycles of stress, *Proc. Amer. Soc. Test. Mat.* **50,** 649–665, 1950.

In order to avoid unnecessary size and weight in the design of some components in aircraft, the critical stress is based on a life less than 50,000 cycles. Authors determine the characteristics of the short-life high stress portion of the S-N curve for four steels, which were heat-treated to 200,000 psi tensile strength. The axial-loading tension and flexural fatigue tests were performed with unnotched (smooth) specimens at the stress range $R = 0$, i.e., from zero stress to maximum tensile stress. The speed in the axial-loading or cantilever bending-fatigue testing machine was 50 or 35 cpm. The tests were made for the range of life $^1/_2$ (axial-loading) or 1,500 (bending) to 50,000 cycles at loads of 80 to 100% of the static tensile strength.

Results of authors' and foreign tests are plotted in terms of life vs. maximum stress in percentage of static tensile strength. By this means, it is possible to compare different materials, test methods, and heat treatments. The short-life portion of the S-N curve shows that fatigue life in pulsating tensile fatigue tests at stress above 80% of the tensile strength is greater than would be expected by extrapolation backward of the long-life portion of the S-N curve. The stress concentration factor is dependent on the lifetime, i.e., this factor becomes less as the test stress increases and the fatigue life decreases.

Results from similar tensile fatigue tests on steels, reported in *Arch. Eisenhüttenw.*, **21,** 1/2, 67–76, 1950, by A. Pomp and M. Hempel, confirm the above conclusions.

M. Hempel, Germany

3881. Clark, D. S., and Wood, D. S., The influence of specimen dimension and shape on the results in tension impact testing, *Proc. Amer. Soc. Test. Mat.* **50,** 577–585, 1950.

Paper deals with the influence of length and shape of cross section of specimen on detection of a "critical" impact velocity at which total energy absorbed to fracture reaches a maximum. Experimentally determined critical velocity for SAE 1020 steel coincided with computed critical velocity based on static stress-strain diagram. Shape of specimen cross section did not influence effect of velocity within dimensions studied. Authors conclude that ratio of length to diameter must be at least 13 to allow critical velocity to be clearly indicated in tests.

G. M. Sinclair, USA

3882. Zapffe, C. A., and Worden, C. O., Fractographic registrations of fatigue, *Trans. Amer. Soc. Metals* **43,** 958–969, 1951.

Fractographic studies of fatigue fractures of 75S-T6 aluminum alloy, mild steel, and alloy steel, show two distinctive patterns which characterize conditions of damage from fatigue stressing. In the preliminary stages the pattern is referred to as "platy" and in the final stages as fine lamellar. Corresponding structures were found to extend into the stressed section of the parent metal, becoming less pronounced with distance from the fracture.

Authors correlate these patterns with a mosaic constitution of the grain and a structural reorganization caused by fatigue. They offer a tentative explanation based on the micellar theory of the solid state. They believe that these patterns promise to reveal the fundamental nature of fatigue and, perhaps, provide a metallographic technique for the recognition of the stages of fatigue. E. B. Shand, USA

3883. Chalmers, B., and Martius, U., Dislocations in real crystals, *Nature Lond.* **167,** 4252, 681–682, Apr. 1951.

In a given crystal, the dislocation of lowest energy is postulated to be that for which the ratio of the unit slip distance (Burgers vector) to the distance between slip planes is least. In every case tested, the slip direction and plane so selected agree with that observed in plastic deformation.

[Reviewer's note: See also Frank, F. C., *Physica* **15,** 1–2, 131–132, April 1949.] F. R. N. Nabarro, England

3884. Mikhalapov, G. S., Evaluation of welded ship plate by direct explosion testing, *Weld. Res. Suppl.* **16,** 4, 195s–201s, Apr. 1951.

Since previous investigations have shown that the explosion test can show the effects of notches and other stress raisers in prime plate, it is assumed that this test can also be used to study the notch sensitivity of welded joints. An investigation was carried out to determine whether variation in welding procedure and welding electrode had an appreciable effect on the notch sensitivity. Two types of ship plates were tested: ABS Class B (semikilled), and ABS Class C (silicon-killed with aluminum added). Two types of manual electrodes were used: Class E-6010, and an alloy electrode falling into Class E-10016. In addition, a submerged arc process, using a standard 2% Mn electrode, was used. All welds were radiographed. All failures were of the cleavage type. In comparison with prime plate, the specimens welded with E-6010 electrodes absorbed much lower energy to

fracture. There was a suggestion that the use of fully killed steel does little to improve the load-carrying capacity as long as conventional welding procedures are used. In comparison with the performance of the E-6010 electrodes, that of the same plates welded with E-10016 electrodes was very much superior. The improvement produced by the submerged arc process is greater at room temperature and extends down to as low as −40 F.

Marshall Holt, USA

3885. Kahn, N. A., Imbembo, E. A., and Ginsberg, F., Effect of variations in notch acuity on the behavior of steel in the Charpy notched-bar test, *Proc. Amer. Soc. Test. Mat.* **50,** 619–647, 1950.

Presents data on effects of variations in notch root radius (from 0.006 in. to 0.021 in.) of Charpy V-notch specimens and of variations in hole diameter (Nos. 46, 47, and 48 drills) of key-hole notches on energy-absorption level and transition temperature in impact tests. Four steels (Navy medium, high tensile strength, and SAE 1045 and 4620) were studied at strength levels from 60,000 to 110,000–155,000 psi. Authors conclude, from results observed, that tolerances should be specified for root radius of such impact test bars.

Polishing the base of ASTM standard 0.010-in. radius V-notch was found to produce little effect on impact values.

Horace J. Grover, USA

3886. Faupel, J. H., and Marin, J., Tension-compression biaxial plastic stress-strain relations for aluminum alloys 24S-T and 2S-O, *Trans. Amer. Soc. Metals* **43,** 993–1012, 1951.

Working on two aluminum alloys 24S-T and 2S-O of a fixed tubular shape, authors obtain experimental data of plastic stress-strain relations of tension-compression biaxial stresses. Results are reported for different constant ratios of the two principal stresses as well as for variable ratios. Problem is to predict biaxial stress-strain relations by simple tension tests. Methods used are simple and convenient for engineering practice.

Testing machine and shape of specimen are described first. True stress-strain relations are calculated from the corresponding measured relations of nominal stresses and nominal strain. Using deformation theory, generalized stresses are plotted against generalized strains, and for all stress ratios give approximately the same stress-strain relation. Also, deformation theory is a sufficient approximation in the case regarded and problem is solved: biaxial stress-strain states may be predicted from simple tests.

Failure by yielding is in good agreement with distortion-energy theory; maximum stress theory as well as maximum shear theory do not give a good approximation to experimental data.

By restricting themselves to simple methods, authors give a good procedure for engineering practice, and here lies the value of the paper. Reviewer regrets that, like most modern papers, it presumes knowledge of references quoted.

Franz Wever, Germany

Design Factors, Meaning of Material Tests

(See also Rev. 3895)

3887. Schwartz, H. A., and Bock, W. K., Reproducibility of results of tension tests on specimens prepared from cast steel coupons, *Proc. Amer. Soc. Test. Mat.* **50,** 1167–1175, 1950.

Data are recorded showing the degree of reproduciblity of the usual static tensile properties of grade B steel as determined on test coupons of several designs. The results of single tests or the average of a small number of tests is shown not to be an adequate measure of the mechanical properties of a given steel cast in a given form of coupon. From authors' summary

3888. Wierzbicki, W., The meaning of chance in the problems of strength of materials (in Polish), Jubilee book edited in honor of Prof. M. T. Huber, Gdańsk-Politechnika, 435–452, 1950.

Because of many unknown factors, chance affects each result in a strength-of-material test, and hence, calculations for strength contain inevitable errors. In contrast to the subjective manner in which designers have handled the element of chance in strength-of-material problems, author introduces his method of objective determination of safety factors, based on the calculus of probability and on mathematical statistics. To describe numerically the gradient of safety, such as to insure a given structure, author bases his theory on a value p which expresses the probability that the structure will not fail (index of safety). The value p should be determined from statistics on failures of similar structures or by comparison with the probabilty of such events as fires, floods, etc.

Author determines the factor of safety n based on the index of safety p from the equation $\Omega_1 \cdot \Omega_2 \cdot \Omega_3 = p$, which is valid for cases of large probabilities, where Ω_1 is the probability that the hypothesis, on which the calculation is based, is true; Ω_2 is the probability that dimensions and leads assumed in calculations are true; Ω_3 is the probability that the strength of a given material will not be exceeded. The probabilities Ω_1, Ω_2, Ω_3, here regarded as real probabilities, are determined by means of Gauss' curve or from tables of Laplace's function. Author gives some examples of determining the values n and p and also shows the advantages of using this method to determine the allowable working shear stress in steel structures [see AMR **4,** Rev. 1579], and also for calculating the cable in suspension bridges [see *Publ. int. Assoc. Bridge Struct. Engng.*, 1949]. K. Zarankiewicz, Poland

Material Test Techniques

(See also Revs. 3746, 3859, 3861, 3884, 3901)

3889. Hartman, E. C., Falling-weight impact test of welded aluminum alloy plates, *Weld. Res. Suppl.* **16,** 6, 303s–306s, June 1951.

Author describes equipment and procedure for a dropping-weight impact test for plate specimens. Specimen is octagonal, 16 in. across the flats, is laid over the circular opening of an anvil $11\frac{3}{4}$-in. diam, and is struck in the center by a round nose tup on a weight. Results are expressible in terms of height of drop, or in.-lb. in blow to produce fracture. Type of fracture as well as energy absorbed are the criteria for comparing performance of materials, welds, or effect of temperature. Test is recommended by author as compromise between small impact tests and full-scale drop tests for shock resistance in pressure-vessel field.

Henry A. Lepper, Jr., USA

3890. Takenaka, J., Research on the knife edges used for testing machines (in Japanese), *Trans. Soc. mech. Engrs. Japan* **17,** 57, 110–117, Apr. 1951.

Paper describes the results of experimental research on knife edges used for testing machines. The first part deals with pressure tests, where the elastic and plastic deformations due to compressive loads and the sustaining power of knife edges of various forms and materials are investigated. The result shows that a knife edge of angle 90° and Rockwell hardness C 62 or more can sustain a specific load (compressive load per unit length of the edge) of 5 ton/cm, and the power becomes a little larger when the

angle is 120°. But in practical designs, a knife edge of 90° is recommended for use under the load 3 ton/cm, considering the impact effect of loads and the facility of manufacture. For the material, hardened high-carbon steel (0.8% C) is as good as high-carbon chromium steel.

The second part deals with friction tests, where the frictional moments of the knife edges, subjected to permanent deformations under various loads, are measured by a pendulum mechanism. The frictional moment increases about 50% as the load becomes 3 ton/cm from no load. Teruyoshi Udoguchi, Japan

3891. Rocha, M., and Borges, F., Photographic method for model analysis of structures, *Proc. Soc. exp. Stress Anal.* **8, 2,** 129–142, 1951.

A method of model analysis as developed and used in the Civil Engineering Laboratory, Lisbon, is described. Principles of method are based on Maxwell-Betty's reciprocal theorems. The main difference is that displacements recorded are measured on a photographic plate, unlike the use of direct visual technique as in methods evolved by Gottschalk, Beggs, and Magnel. Authors' method is claimed to give quicker and easier results, besides supplying a documentary evidence of test, which can always be relied upon for future use. Authors describe technique and give tabular comparisons with Beggs' and analytical methods in case of ordinary single bents, arches, and complex theater frame.

Developed on correct lines, this photographic method appears to have wide potentialities in model structural analysis.

S. K. Ghaswala, India

3892. d'Entremont, B. H., Nondestructive testing of steels (in French), *Metallurg. Constr. mécan.* **83,** 2, 3: 97–105, 161–167; Feb., Mar. 1951.

This is a review paper on nondestructive testing, divided into three groups: I. Sorting methods; II. Methods of detecting internal defects; and III. Experimental methods. Under I are considered hardness, thermoelectric effects, spark tests, magnetic tests. Various means of detecting defects discussed under II are hardness variations, chemical attack, crack detection (Zy-glo, etc.), magnetic methods, ultrasonics, and radiography. Experimental methods include measurements of elasticity, resonance-vibration techniques, and measurements of stress-concentration effects by stress-coat and photoelasticity.

Louis F. Coffin, Jr., USA

Mechanical Properties of Specific Materials

(See also Revs. 3846, 3847, 3849, 3867, 3868, 3871, 3872, 3874, 3877, 3878, 3879, 3880, 3881, 3885, 3886, 3999)

3893. Andrade, E. N. da C., and Kennedy, A. J., A surface effect in the creep behaviour of polycrystalline lead, *Proc. phys. Soc. Lond. Sec. B,* **64, part 4,** 376 B, 363–366, Apr. 1951.

We know that, in polycrystalline wires, transient creep is associated with glide within the crystal grains, while permanent creep is associated with intergranular movement. Glide within the grains is influenced by surface conditions and in a polycrystalline material the grains at the surface are less constrained than those well in the interior. Hence, surface layers should offer relatively small resistance to transient flow.

Authors made tensile creep tests with wires of pure lead with 0.05% tellurium added. This material shows only transient creep at room temperature, except at the highest stresses. It has been found that to produce the same transient creep in different wires requires an average stress which increases with the thickness of the wire. This proves that surface layers offer relatively small resistance to transient creep. This nonuniformity of behavior over the cross section has been further established by x-ray examination. Aris Phillips, USA

3894. Neurath, P. W., and Koehler, J. S., The plastic deformation of pure single crystals of lead and copper, *J. appl. Phys.* **22,** 5, 621–626, May 1951.

Paper deals with creep tests of pure single crystals of lead and copper. Test procedure and specimen preparation are described. The step-loading creep results of copper crystal at 0 C, and of lead crystal at 28 C and 190 C are reported. Authors try to determine the resolved shearing stresses below which the steady-state creep is negligible for copper and lead. It is interesting to notice that the reported transient creep in pure copper crystal at 23 C under a resolved shearing stress of about 800 g/mm² agrees with C. L. Smith's derivation, namely: *Creep strain = constant* × log (*constant* × *time*). Ling-Wen Hu, USA

3895. Marin, J., Creep rates of acrylic plastics, *Prod. Engng.* **22,** 7, 171–175, July 1951.

Creep data similar to that published in *Trans. Amer. Soc. mech. Engrs.* **73,** 5, 705–719, July 1951, is presented here in a form more suitable for design engineers. Yoh-Han Pao, USA

3896. Manjoine, M. J., Effect of rate of strain on the flow stress of gas turbine alloys at 1200 and 1500 F, *Proc. Amer. Soc. Test. Mat.* **50,** 931–948, 1950.

Creep, creep-rupture strain rate, and high-speed impact test results are given for ten high-temperature alloys.

J. C. Fisher, USA

3897. Keylwerth, R., Fillet section in wood tension members and tension-test specimens (in German), *Bauingenieur* **26,** 2, 42–44, Feb. 1951.

An exponential formula is derived and used for the design of fillet sections of wood tension members and tension-test specimens. Its validity is proved by experimental data for air-dry pine. Consideration is given also to the design of the head section of test specimens for fastening it with gripping wedges in testing machines. E. George Stern, USA

3898. Liska, J. A., Methods of calculating the strength and modulus of elasticity of plywood in compression, *For. Prod. Lab. Rep.* 1315, 4 pp., Dec. 1950.

Mathematically, only approximate (however practically satisfactory) formulas are presented which are applicable to plywood with perpendicular grain of adjacent plies, loaded parallel or perpendicular to grain direction, whenever buckling is not a controlling factor. E. George Stern, USA

3899. Hillier, K. W., The variation of the dynamic elasticity of polyethylene with strain, *Brit. J. appl. Phys.* **2, 6,** 167–170, June 1950.

Variation in the dynamic elastic modulus of polyethylene filaments subjected to increasing tensile strain is reported for several temperatures between 10 and 54 C. The dynamic modulus was determined from values of the velocity of propagation of longitudinal oscillations at frequencies of 2 to 16 kc/s. Outstanding features of results, according to author, are (a) a large increase in dynamic modulus with strain compared to a small decrease in "static" modulus, and (b) a minimum in the dynamic modulus vs. strain curve and a point of inflection in the static stress-strain curve. A discussion of the results is given, based on the structure

of the filaments and the probable changes caused by stretching.
B. M. Axilrod, USA

3900. Hussey, F. W., Rockwell C hardness on cylindrical steel specimens, *Proc. Amer. Soc. Test. Mat.* **50,** 1176–1185, 1950.

A series of somewhat more than 2800 individual hardness readings were made on the flat and curved surfaces of carefully prepared steel specimens of small diameter to determine the deviations of hardness readings taken on curved surfaces from the "true" values of flat areas. Eighty steel specimens of eight diameters were turned down from larger stock to assure homogeneous metal, heat treated to obtain ten ranges in hardness in each size group, and then ground to size with a fine degree of surface finish. The hardness tester was operated and calibrated in accordance with the practices recommended in ASTM Standard Methods (E 18-42) except that even stricter control was exercised over some variables. The resultant deviations, when plotted, indicated line relationships of such slight curvatures that straight line averages, obtained by the method of least squares, were utilized and considered sufficiently accurate for all practical purposes.

Correlation curves are presented which may be used to obtain actual Rockwell C hardness values from measurements made on cylindrical surfaces of the following diameters: 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1, and 1 1/4 in. From author's summary

3901. Rondeel, J. H., Mechanical properties of core materials of sandwich construction (in Dutch), *Plastica* **4,** 3, 74–78, Mar. 1951.

Determinations of Young's modulus and modulus of rigidity of calcium alginate solid foam, vulcanized expanded rubber (Onazote) and honeycomb construction (Dufaylite) of English manufacture are described.

Young's modulus was measured in tension and the modulus of rigidity was calculated from results of three-point bending tests. Results are in general agreement with those reported by English workers. F. T. Barwell, Scotland

3902. Weil, L., and Bochirol, L., Measurement of Young's modulus of ferrites (in French), *C. R. Acad. Sci. Paris* **232,** 20, 1807–1809, May 1951.

The value of Young's modulus of a ferrite mixture $Fe_2O_3Ni_{0.3}Zn_{0.7}O$ has been determined by exciting a radial resonance in a ring of the material. The saturation magnetostriction constant of one part in 10^6 is sufficient to drive the resonance. The measured value of 1.17×10^{12} dyne/cm^2 for the ring having a density of 4.5 is less than that for a single crystal having a density of 5.29 as measured by x rays. Warren P. Mason, USA

3903. de Haller, P., Erosion and cavitational erosion, *David W. Taylor Mod. Basin Transl.* **188,** 27 pp., Apr. 1951.

Translation from *Handbuch der Werkstoffprüfung,* **2,** 471–488, 1939.

3904. McElhinney, D. M., The effect of grain direction on mechanical properties of light alloy extrusions, *Aircr. Engng.* **23,** 265, 62–66, Mar. 1951.

Paper deals with anisotropy of large extruded bars of aluminum aircraft alloys as evidenced by differences in tensile strength and elongation of rectangular specimens cut from various locations and at different orientations in bar. In longitudinal direction, ultimate strength of specimens gradually decreases from edge of bar toward center. Loss of strength due to transverse grain is found to be much greater than assumed in past, being in some cases as great as 35%. With stress concentration greater than 2, reduction in tensile strength is as much as 45%. Izod impact values are reported in addition to results of more than 200 tensile tests of notched and unnotched specimens. For design purposes, author makes recommendations for estimating loss of strength in large extruded bars and large forgings according to grain direction. Data are expressed in British units.
G. M. Sinclair, USA

3905. Malstrom, C., Keen, R., and Green, L., Jr., Some mechanical properties of graphite at elevated temperatures, *J. appl. Phys.* **22,** 5, 593–600, May 1951.

Measurements of tensile strength, creep, and Young's modulus were made on graphite specimens at various temperatures, in connection with the interest in stressed ceramic parts for service at very high temperatures. All specimens were fabricated with their axes parallel to the direction of maximum strength.

Tensile-strength data are reported for five commercial grades of graphite. The specimens were loaded through ball-bearing universal joints and heated by their own electrical resistance. The tests were conducted in a helium atmosphere. Temperatures were measured with an optical pyrometer, sighted through windows in the furnace. No information is given on the rate of stressing.

The strength of all graphites increases with increases in temperature to about 2500 C, and then abruptly decreases. At 2500 C, the strength of each graphite is about twice the room-temperature value. Failure at all temperatures is by brittle fracture. The density of the graphite specimens of each grade varies, depending on the particular location in the molded block or extruded rod from which they are taken. In tests on one grade, a linear relationship is found between strength and density at each testing temperature. However, variations in strength between different grades of graphite could not be correlated with density or with any other apparent factor. Authors feel that the abrupt decrease in strength at 2500 C results from the beginning of sublimation and of rapid creep.

No changes are noted in grain orientation or room-temperature strength when specimens are heated and then cooled. The similarity with the strength-temperature behavior of sillimanite porcelain is noted by authors.

As such behavior is not uncommon in ceramic ware, reviewer feels the explanation might lie in common structural characteristics. All of these materials consist of inclusions, a bonding phase, and pores. Internal stresses in the inclusions and in the bonding phase would decrease with increases in temperature until the temperature at which the stresses had been "frozen in" is reached. The size of the pores also would be temperature dependent, and the stress-concentrating action of a pore varies with its size. Hence, the magnitude of stress peaks in minute volumes of the graphite is a function of temperature. Their magnitude also varies with the external stress. With fracture occurring when a stress peak reaches a critical magnitude, it seems logical that increases in temperature cause such a decrease in dangerous stress peaks as to require an increased external stress for fracture.

Creep measurements were made with essentially the same equipment as used to obtain the tensile-strength data. Elongation was measured with a micrometer microscope sighted on the image of the specimen on a screen. No creep data are reported at temperatures below 2100 C. Curves of elongation versus time are qualitatively similar to those characteristic of metals. A steady creep rate is preceded and followed by transient, increasing rates. With reservations, authors feel that the data might be interpreted in terms of Eyring's rate-process theory. There is evidence that a different mechanism governs the creep rate at temperatures above about 2500 C than at lower temperatures.

Young's modulus is measured dynamically from the frequency

of resonance vibration of a cantilever-beam specimen. Apparently, measurements are made on only one grade of graphite at temperatures in the range from 1000 to 2000 C. Static measurements are made on the same specimens at room temperature. The Young's modulus increases almost linearly from about 1.0×10^1 psi at room temperature to about 1.4×10^6 psi at 2000 C. Authors could not explain this behavior. Again, reviewer feels an explanation might lie in a consideration of structural characteristics. The influence of temperature on the various phases and pores would be expected to alter the gross effect of stress peaks; hence, Young's modulus.

Authors point out several possible sources of experimental error, and consider lack of homogeneity in the graphites as the most serious. Reviewer feels that inhomogeneity is characteristic and, particularly in fracture data, must be given statistical consideration. Winston Duckworth, USA

Mechanics of Forming and Cutting

3906. Oehler, G., Deep-drawing quality of sheet metal. Cup-drawing test and bulging method (in German), *Z. Ver. dtsch. Ing.* **93,** 13, 371–374, May 1951.

Historical review of and criticisms on the different methods of testing materials by either the cup-drawing test or the bulging test. In the first, the deformation is a combination of stretch in one direction and of compression in the second direction. In the bulging test, both components of the deformation are elongations. The two methods lack, therefore, correlation and the cup-drawing test, being closer to practical use, is preferred. Need for standardization is emphasized and the numerical data of proposals to this effect made in Germany are given. C. Zwikker, Holland

3907. Fersing, L., Carbide high-velocity turning, *Trans. Amer. Soc. mech. Engrs.* **73,** 4, 359–374, May 1951.

Tool forces are measured by dynamometer for steels SAE-C-1118 and SAE-8747 for cutting speeds up to 1400 fpm and various rake angles and feed. Average chip temperatures are obtained by calorimeter method. Photographs of the cutting process are included together with some examples of production applications. R. N. Arnold, Scotland

Hydraulics; Cavitation; Transport

(See also Revs. 3923, 4039)

3908 Escande, L., Supplementary problems in hydraulics. 2nd part (Compléments d'hydraulique. 2me partie) Paris, Dunod, 1951, 248 pp., 162 figs. Fr. 1900.

Companion volume to one published in 1947 [AMR **2,** Rev. 1152], differs in binding, size, and type of reproduction, but again discusses series of loosely related problems from author's consulting and laboratory practice, many already published elsewhere. Nine chapters deal with following topics: Water-hammer effects during simultaneous variation of three discharge controls [see AMR **3,** Revs. 285 and 1298]; overpressure resulting from closure of butterfly valve by servomotor [AMR **2,** Rev. 75]; maximum pressure during regulation of hydroelectric systems equipped with discharge valves [AMR **2,** Rev. 789]; influence of inlet constriction on surge-tank performance [AMR **3,** Rev. 2686]; graphical analysis of surge-tank performance for ports of small cross section [AMR **1,** Rev. 732]; oscillations in connecting reservoirs; sector gates on spillway crests [AMR **3,** Rev. 1212]; spillways with simultaneous overflow and underflow [AMR **3,** Rev. 716]; similitude considerations, and flow regulation. Although reproduced from typewritten copy, the text and numerous diagrams are legible and useful. Hunter Rouse, USA

3909. Eisenberg, P., and Pond, H. L., Water tunnel investigations of steady-state cavities, *David W. Taylor Mod. Basin Rep.* 668, 19 pp., Oct. 1948.

This is the second report [following *David W. Taylor Mod. Basin Rep.* 647] on a study aimed at determining empirically the shape of bodies of revolution having specified critical cavitation numbers by measuring cavitation bubbles formed behind disks and other obstacles in a stream of water. Reichardt, who first used this method in Germany, obtained smooth-walled cavities by introducing air behind the obstacle. The vapor cavities obtained in these tests, however, are unsteady and rough-textured, although they appear smooth on long-exposure photographs.

Pressure-distribution measurements along the axis of the cavity show greater variations in pressure than can be explained if the cavity is assumed to be filled with vapor. The deviation of the internal pressure from vapor pressure diminishes as the cavitation number is lowered. This suggests that at the higher cavitation numbers this may be a flow with a water wake and with cavitation in the small scale eddies in the boundary of the wake. Author suggests further experiments with sampling of the fluid and velocity and temperature surveys.

Measurements of the resultant drag force are in agreement with extrapolation made from tests by Reichardt and others, using the formula

$$C_D(\sigma) = C_D(0)(1 + \sigma),$$

where $C_D(\sigma)$ is the drag coefficient at cavitation number σ, and $C_D(0)$ is the drag coefficient for cavitation number zero. Joseph Levy, USA

3910. Escande, L., On overtopping of rockfill dams (in French), *C. R. Acad. Sci. Paris* **232,** 15, 1402–1404, Apr. 1951.

Tests have been made on a 1:80 scale model of a rockfill dam 20-m high, made of rocks varying in size from 16 to 32 cm to study the behavior under overtopping conditions. Rapid failure occurred when the downstream face of the dam was not protected. Tests were repeated with concrete blocks representing $1 \times 1 \times 1.75$-m blocks on the prototype protecting the downstream face. Rapid infiltration of the water caused an uplift pressure which lifted the blocks, destroyed the rip-rap, and finally the dam. Drains were then installed under the dam to eliminate uplift and the dam did not fail, but the importance of the drainage required makes this solution impossible on the prototype.

Author concludes that overtopping should not be allowed on rockfill dams, even when protected by dry rubble on the downstream face. Author suggests a solution consisting of a protective facing on the downstream slope, which will be flexible enough to follow the settlements of the dam without cracking and also impervious enough to prevent rapid infiltration of the water in the dam. Author does not state how such a facing could be designed. André Leclerc, Canada

3911. Durand, R., Different technical definitions of pebble shape (in French), *Houille blanche,* **6,** no. A, 253–261, May 1951.

Author considers in the group of shape coefficients the non-dimensional "intrinsic" coefficients and those which are peculiar to each technique. The intrinsic coefficients are divided into three categories, depending on whether the sizes are measured according to a direction, a plane, or in space. "Sphericity" and "roundness" can be characterized by the use of a graph of moduli, similar to those defined by concrete technicians. As far as the co-

efficients peculiar to each technique are concerned, author develops more particularly the hydraulic engineer's viewpoint: flow in porous media, behavior of rock massifs, velocity of transportation, and velocity of fall. G. A. T. Heyndrickx, Belgium

3912. Bagnold, R. A., Measurement of very low velocities of water-flow, *Nature* **167,** 4260, 1025–1027, June 1951.

Paper describes new instrument for accurately measuring velocities as low as 0.2 cm/sec in flowing water. Principle involves balancing flow of water through small orifice in tip of instrument against a counterflow of electrolyte of the same density as water. Balance is determined electrically. Instrument may be used successfully in streams containing suspended solids or may be buried in sand beds to measure seepage.

W. M. Owen, USA

3913. Escande, L., Inefficiency of surge tanks in case of certain rhythmical overpressures (in French), *C. R. Acad. Sci. Paris* **232,** 9, 790–792, Feb. 1951.

3914. Zicman, B. R., On the calculation and dimensioning of surge tanks (in Spanish), *Cienc. y Técn.* **116,** 583, 27–40, Jan. 1951.

Review of numerical and graphical methods for the calculation of surge tanks of different types; the making of models is also considered. Duilio Citrini, Italy

3915. Binnie, A. M., Theory of waves travelling on the core in a swirling liquid, *Proc. roy. Soc. Lond. Ser. A,* **205,** 1083, 530–540, Mar. 1951.

Author continues his analysis of liquid flow through a nozzle [AMR **2,** Rev. 198; **3,** Revs. 2369, 2421]. Because of swirl, a hollow core is formed, whose surface is disturbed by progressive waves. Their form may be predicted considering irrotational motion in a cylindrical pipe: $\phi = -\Omega\theta - F(r) \sin(mz + n\theta - \sigma t)$, F being small. The waves are of varicose form with circular cross section of the core ($n = 0$), or helical, giving the core the shape of a multithreaded screw ($n > 0$). The relation between the length $2\pi/m$ of a wave of order n and its axial (σ/m) and angular (σ/n) velocity is found, taking into account centrifugal force and surface tension. Also, the limiting case of vanishing surface tension is examined. Results are extended to pipe of infinite diameter (tornadoes). Propagation upstream of waves originating in a throttle is discussed. Group velocities are calculated. Gino Moretti, Argentina

3916. Mooney, M., The viscosity of a concentrated suspension of spherical particles, *J. Colloid Sci.* **6,** 2, 162–170, Apr. 1951.

Einstein's equation relating viscosity of an infinitely dilute suspension of rigid spheres to the volume concentration of the dispersed phase is extended to apply to a suspension of spheres of finite concentration by considering the crowding effect between equisized spheres and between spheres of different diameter. For equisized spheres the equation $\eta_r = \exp(2.5\phi/(1 - k\phi))$ is developed where η_r is the relative viscosity of the suspension, ϕ is the volume concentration of the dispersed phase, and k is a constant, the self-crowding factor, the value of which is estimated on a theoretical basis. Equations are also given for suspensions of spheres of two different diameters and for a more general case of a suspension of spheres covering a well-defined size range.

In developing the theoretical part assumptions are made, the validity of some of which may be open to question, although there appears to be good agreement of the theory with published experimental data. There are one or two misprints in the equations. Stacey G. Ward, England

3917. Valken, K. F., Graphical method to draw backwater curves for stationary flows in open watercourses (in Dutch), *Ingenieur* **63,** 23, B.73–B.78, June 1951.

Paper develops a graphical method for the integration for Euler's equation of motion in one-dimensional approximation for flow in an open watercourse. Taking as independent variable the water height and neglecting the velocity term in first approximation, a simple first-order differential equation is obtained for which graphical method is easily found.

The complete equation is treated as a generalization of this first case and is seen to require little additional work. Method is easily adapted to different water depths, rates of flow, or bottom slopes. R. Timman, Holland

Incompressible Flow: Laminar; Viscous

(See also Revs. 3757, 3917, 3943, 3944, 3956, 3957, 3982, 3991, 4023)

3918. Streeter, V. L., Fluid mechanics, New York, London, Toronto, McGraw-Hill Book Co., Inc., 1951, viii + 366 pp. $5.

Book represents an introductory course in fluid statics and fluid flow. The first two chapters deal with fluid statics, followed by a chapter introducing the concepts, basic definitions, and equations of fluid motion. Another chapter is devoted to basic principles of turbomachinery with application to pumps, blowers, turbines, fluid coupling and torque converters, with a brief discussion of the basic principles of jet propulsion.

Considerable attention is given to flow in tubes, conduits, and channels. Flow around immersed bodies is discussed in a separate chapter covering simplified treatments of equations of motion, velocity potential, stream function, boundary layer, and drag. A final chapter deals with oil hydraulic systems.

In addition to the usual type of numerical problem, a number of objective problems of the multiple-choice type are given at the end of each chapter so that the student may derive increased benefit from the two different approaches.

Frank L. Wattendorf, USA

3919. Dzhanelidze, G. Yu., On the theory of thin or thin-walled bars (in Russian), *Prikl. Mat. Mekh.* **13,** 597–608, 1949.

Paper presents a generalized theory of elastic deformation of thin-walled cylinders of uniform cross section. The strain equations are taken in the form $e_z = e_y = \gamma_{xy} = 0$,

$$\gamma_{xz} = (\partial\theta/\partial z)(\partial\phi/\partial x - y), \quad \gamma_{yz} = (\partial\theta/\partial z)(\partial\phi/\partial y + x),$$
$$e_z = e + \nu_1 y - \nu_2 x + \phi(x, y)\partial^2\theta/\partial z^2),$$

where the terms not found in the classical treatment of the problem are ν_1 and ν_2, which are the curvatures of the axis of the shell, and $\partial^2\theta/\partial z^2$ the derivative of the angle of twist. Energy expressions are then used to obtain the formulas for the applied moments and forces as well as for the natural boundary conditions. These formulas take account of the interaction between torsion, bending, and extension which is neglected in the classical formulas. Equations of motion of a cylindrical shell in combined torsion and flexure are derived by use of Hamilton's principle. They differ from the corresponding classical equations by a term in the fourth derivative of the angle of twist. Results of the present theory are shown to be a generalization of Vlasov's work on thin-walled cylinders with open contours and Umanski's work on closed cylinders.

Courtesy of Mathematical Reviews. H. I. Ansoff, USA

3920. Arzhanikh, I. S., A vortex interpretation of the theory of functions of a complex variable (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **73,** 667–670, 1950.

Let $F(\zeta) = \varphi + i\psi$, $\zeta = x + iy$, be an analytic function interpreted as the complex potential of a steady incompressible flow. Author observes that this flow may be transformed into a rotational three-dimensional flow by assuming a z-component of the velocity vector $\theta(\psi)$, where θ is an arbitrary function. The vorticity $(\Omega_x, \Omega_y, \Omega_z)$ of this flow is given by $\Omega_x - i\Omega_y = F'\theta'$, $\Omega_z = 0$. The stream surfaces are given by (1) $\psi = \text{const}$, (2) $z = \theta(\psi)\int\{|d\zeta/dF|^2 dF\}_{\psi=\text{const}} + \omega(\psi)$ where ω is an arbitrary function. Author calls (1) the equation of the wing and (2) that of the fuselage. For a given $\psi(x, y) = a$ an equation $z = V(x, y) + c$ will represent a possible fuselage if and only if (!) $(\partial/\partial(x, y))\{\partial(V, \psi)/\partial(x, y), \psi\} = 0$. The totality of all fuselages obtained in this way is described by a complicated partial differential equation obtained from (!) by eliminating ψ by means of Laplace's equation. Similar considerations are carried out for the unsteady case, $F = F(\zeta, t)$. L. Bers, USA

3921. Truckenbrodt, E., Supplements to F. Reigels: Incompressible potential flow past given obstacles (in German), *Ing.-Arch.* **18,** 5, 324–328, 1950.

In two earlier papers [AMR **3,** Revs. 1507 and 2393] Riegels proposed a method to improve the familiar theory of thin airfoils in plane incompressible flow. His approximation consists essentially in a correction for finite thickness which would be exact in the case of an elliptic cylinder. To complete this work, the present author transforms Riegels' formulas for aerodynamic coefficients into sums over the profile ordinates with constant coefficients, tabulated here. In Riegels' second paper the equivalent coefficients were calculated in terms of Fourier coefficients, which in turn were calculated from the ordinates.

W. R. Sears, USA

3922. Schlichting, H., and Scholz, N., On the theoretical calculation of flow losses of a plane cascade (in German), *Ing.-Arch.* **19,** 1, 42–65, 1951.

Losses of two-dimensional cascades are calculated by finding the pressure and velocity distributions on the airfoil surfaces from the potential flow solution and then generating boundary layers on these surfaces from laminar and turbulent boundary-layer theories. The change in the exit flow direction caused by these boundary layers is also calculated. Profile shapes are generated by an elliptic chordwise distribution of circulation, and have zero thickness.

Loss coefficients are evaluated for eight different cascades (compressor, turbine, and equal pressure) but no experimental comparisons are offered. The compressor cascades investigated show separation on the concave surface due to the unusual profile shape. Authors indicate the need for extensive calculations to include airfoils of finite thickness, different profile shapes, and a range of stagger and incidence angles. R. E. Kronauer, USA

3923. Craya, A., Critical regimes of flows with density stratification, *Tellus* **3,** 1, 28–42, Feb. 1951.

Author discusses critical flow criteria for both internal and surface gravity currents and shows analogy to the case of simple open-channel flow. The analysis is quite clear. Discussion concerns, first, open-channel flow, then homogeneous gravity currents and, finally, currents with density stratification. In the latter case the calculus of variations is used for minimizing.

For currents with density gradients, author shows the significant difference in the critical flow criteria, depending on whether the energy or momentum theory is used. For surface currents, if the vertical displacements are restricted, maximum velocity will occur at some distance down from the surface. Thorough discussion covers both theory and results.

Harlow G. Farmer, USA

3924. Pierson, J. D., On the virtual mass of water associated with an immersing wedge, *J. aero. Sci.* **18,** 6, 430–431, June 1951.

Author expresses virtual mass in terms of an integral over boundary values of velocity potential—available from a previous calculation. He obtains variation of virtual mass with deadrise angle between Mayo (above) and Wagner-Sydow (below) results; result agrees well with approximate empirical results.

A. Charnes, USA

3925. Rushton, J. H., Some application of fluid mechanics to mixing, *Proc. nat. Conf. industr. Hyd.* **3,** 119–135, Mar. 1950.

Article describes applications of similitude theory to tank-mixing processes. Flow patterns for axial flow (propeller) and radial flow (turbine) impellers are generally described. Power requirements are given in terms of geometrical configuration of system and dimensionless flow numbers (Reynolds, Froude, Weber) for baffled and unbaffled tanks. Comparison of various impeller types is facilitated by graph showing power-number Reynolds-number relations. A. Shaffer, USA

3926. Aoi, T., On the slow motion of a viscous fluid past a sphere (in Japanese), "Studies in mathematical physics," Iwanami Shoten Publ. Tokyo, **1,** 151–168, 1950.

Using results of S. Goldstein [*Proc. roy. Soc. Lond. Ser. A*, 123, 225–235, 1929] on the viscous flow past a sphere based on the Oseen approximation, author calculates streamlines at Reynolds number (based on diameter) $R = 1$, pressure distributions on the surface for $R = 1, 2$, and the drag of the sphere. From a simplified expression for the stream function it is found that reversal of flow (vortex ring) appears on the rear surface of the sphere even at very small Reynolds numbers. It is also shown that the pressure drag and frictional drag are one and two thirds of the total drag, respectively. Isao Imai, Japan

3927. Pieruschka, E., The mathematical foundations for a measurement method of the shear modulus of viscous fluids (in German), *Z. angew. Math. Mech.* **30,** 8/9, 264–265, Aug./Sept. 1950.

Mathematical investigation of viscous flow occurring in special test equipment used to determine the shear modulus of a viscous fluid. Test equipment has oscillatory motion. Another basic work is that of H. Fromm [title source, **28,** 43–54, 1948].

Data on three of eight fluids investigated yield a negative shear modulus. This being impossible, author attributes discrepancy to another unknown type of fluid flow. Assumptions made in deriving solution of differential equation may also be a source of error. Frank J. Mehringer, USA

3928. Dean, W. R., Note on the motion of liquid near a position of separation, *Proc. Camb. phil. Soc.* **46,** part 2, 293–306, Apr. 1950.

The problem of this paper was suggested to its author by previous work done by S. Goldstein, who solved the boundary-layer equations for flow along a wall or around an immersed cylinder. Two papers on the subject by Goldstein are referred to in the present one. In the first, a general solution of the equations was obtained, but it was pointed out that singularities in the solution at which points it was not valid, occurred at points of separation. The second paper gave a solution of boundary-layer equations in

the neighborhood of a point of separation, assuming the existence of a singularity at the point of separation.

Dean's paper gives a solution of the equations of motion near a point of separation. He does not assume the boundary-layer conditions, but solves the more general equations of motion. The problem considers the steady motion of a viscous liquid of constant density with inertia effect included.

It is assumed that a stream function may be defined and expanded in a double series in the x and y coordinates, parallel and normal to the surface, respectively. Pressure is then given as a power series in x along the y-axis and the x-velocity component is expanded in a power series in y along the x-axis.

It develops that the motion may be determined near the origin and that for certain conditions the origin becomes a point of separation of the flow which divides along a straight line of the form $Ay + Bx = 0$. The results of the work are compared with Goldstein's results from the boundary-layer theory. Sufficient details of Dean's work are given so that the method used can be followed readily by readers who wish to study his paper more closely. W. W. Hagerty, USA

3929. Nigam, S. D., Rotation of an infinite plane lamina: boundary layer growth: Motion started impulsively from rest, *Quart. appl. Math.* **9,** 1, 89–91, Apr. 1951.

Paper discusses transient behavior when infinite plane in incompressible viscous fluid is suddenly rotated in its plane. The steady-state solution for constant angular velocity is due to von Kármán. Author linearizes equations with respect to time which, with some simple transformations, leads to ordinary differential equations for velocity components and pressure. Analytical solutions are obtained. Boundary-layer growth is proportional to square root of time. Comment is made on anomalous behavior of pressure. Author also solves for stream surfaces. Above linearization precludes determination of time of transient phase. Morton Finston, USA

Compressible Flow, Gas Dynamics

(See also Revs. 3949, 3952, 3955, 3957, 3958, 3972, 3980, 3982)

3930. Sauer, R., Flow of compressible fluids [Écoulement des fluides compressibles], Paris and Liége, Libr. Polytechn. Ch. Béranger, 1951, xvi + 307 pp.

Book deals with mathematical methods, analytical as well as numerical and graphical, that are employed in studying the flow of nonviscous compressible fluids. The scope is broad; both linearized and exact theories are given for a wide variety of steady and unsteady flows. Since the presentation is clear, concise, and retains a relatively elementary character throughout, the book should be valuable not only to the theorist but also to engineers and physicists concerned with practical applications. Volume includes most of the methods in use up to about 1945.

The first chapter discusses the fundamental mechanical and thermodynamical concepts and presents the basic equations of compressible fluid flow. In addition, unidimensional flows in tubes are discussed in detail. Stationary and nonstationary flows with either constant or variable entropy are included. Linearized theory is treated next, including applications to the propagation of pressure waves and to stationary subsonic and supersonic flows about airfoils and bodies of revolution. No mention, however, is made of three-dimensional flows about wings of finite span. The third chapter deals with more rigorous theories for steady supersonic flow and for pressure waves of large intensity. In addition to presenting the classical two-dimensional steady-flow theory, several pages are devoted to the rigorous theory of axisymmetric steady flow. The fourth chapter describes the foundations and methods of the theory of characteristics from the point of view of one interested in practical applications, and cites applications to two-dimensional and axisymmetric steady supersonic flows and to nonsteady flows in tubes. Chapter five, on exact theory of stationary subsonic flow, contains considerable discussion of transonic flows as well. Three classical methods of solution, series, iteration, and hodograph, are described. It is unfortunate that the small perturbation theory of transonic flow was not mentioned, but this may be due to the fact that the apparent five-year delay between writing and publishing marks almost the entire life of this theory. Chapter six is concerned with the fundamental laws of normal and oblique compression shocks and normal detonation shocks. The seventh chapter discusses flows, both steady and unsteady, containing shock waves. The formation, propagation, and interaction of shock waves are treated in the first several sections, followed by discussions of steady supersonic flows about cones, airfoils, and bodies of revolution. The chapter closes with three sections on plane and spherical explosion waves.

Reviewer recommends book to anyone who wants an introduction to mathematical theory of steady and unsteady flows of compressible fluids. John R. Spreiter, USA

3931. Slezkin, N. A., On the differential equations of gas motion (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **77,** 2, 205–208, Mar. 1951.

The equations of continuity, motion, and energy are written in orthogonal curvilinear coordinates differing from the usual form by the addition of terms representing diffusion and thermodiffusion. Thus, for instance, the continuity equation in Cartesian coordinates is

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\left(\rho v_i - D\frac{\partial \rho}{\partial x_i} - \beta\frac{\partial T}{\partial x_i}\right) = 0$$

where the letters have the usual meaning.

Edgar Krahn, England

3932. Bernard, J. J., Thickness of a steady shock wave, *J. aero. Sci.* **18,** 3, p. 210, Mar. 1951.

Author derives perturbation equation for velocity gradient in one-dimensional flow when the Prandtl number P is close to $^3/_4$. Assuming the solutions for large P to be acceptably near to solution for $P = {}^3/_4$, a special case for which the exact solution is known, author determines shock thickness as distance between two points where greatest curvature of velocity distribution occurs. Graph of shock-wave thickness as function of P for several viscosity variations, at Mach number equal to 1.732, is given. Paper represents part of a more comprehensive report by author in French. T. F. O'Brien, USA

3933. Bardsley, O., and Mair, W. A., The interaction between an oblique shock-wave and a turbulent boundary-layer, *Phil. Mag.* (7), **42,** 324, 29–36, Jan. 1951.

Results reported were obtained at a Mach number of 1.965 by taking schlieren photographs. They may be grouped into three categories of shock-wave strength (static pressure ratio across wave, $(p_2 - p_1)/p_1$, where station 1 is upstream and station 2 downstream of the incident shock wave): (1) Oblique shock waves of strength less than 0.4 are reflected as a compression wave followed by an expansion, with no thickening of the boundary layer upstream of the incident wave. Authors point out that this result agrees with Lighthill's theory, in which the turbulent boundary layer is replaced by a nonturbulent inviscid layer with

the same velocity distribution as a turbulent boundary layer. (2) When the strength of the shock wave is more than about 0.4, the boundary layer begins to thicken and separate on the upstream side of the incident wave and the reflected shock wave moves up to this point. Thus the incident and reflected waves cross at a point outside the boundary layer. (3) As the shock-wave strength is increased to 1.2 or greater, a Mach-type reflection occurs with a short normal shock wave between the two intersections.

Reviewer notes that authors' experimental findings for weak shock waves agree with those of Liepmann, Askenas, and Cole [*U. S. Air Force Tech. Rep.* 5667]. Vernon J. Rossow, USA

3934. Gelbart, A., and Resch, D., A method of computing subsonic flows around given airfoils, *Nat. adv. Comm. Aero. tech. Note* 2057, 34 pp., Mar. 1950.

A simple computational method is developed for obtaining circulatory flows around closed two-dimensional profiles of fluid with linear pressure vs. specific-volume equation of state. The transformation from hodograph to physical plane is that of Gelbart [AMR **1**, Rev. 494] or Lin [*Quart. appl. Math.* **4**, 291–297, 1946]. Procedure is to approximate a certain analytic function by minimizing maximum deviation from the actual transformation effecting mapping above. William Pell, USA

3935. Jacobs, W., Theoretical calculation of the downwash factor for unswept trapezoidal wings with regard to the compressibility for subsonic velocities, *Flygtekn. Försöksanst. Medd.* **38,** 20 pp., 1950.

Paper outlines procedure for obtaining downwash factor for unswept trapezoidal wings in terms of aspect ratio, taper ratio, and tail distance. Downwash is computed by Multhopp's method [*Luftfahrtforschung* **15,** p. 463, 1938] and the influence of compressibility introduced by Prandtl rule. Mean downwash factor which is plotted against tail distance for several aspect ratios (3, 6, 10, 15) and taper ratios (1, 0.6, 0.2) indicates a decrease with decreased taper and aspect ratio and a slight increase with increased tail distance behind wing.

For low aspect-ratio cases, the distortion and rolling up of the vortex sheet behind the wing, not accounted for by Multhopp method which assumes a flat sheet, become increasingly important and may have a sizable effect on the downwash distribution at the tail lengths considered. [See paper by Spreiter and Sacks, AMR **4**, Rev. 2129.] For these cases, it may be necessary to introduce a correction to the downwash factors computed in this paper. Seymour Lampert, USA

3936. Tomotika, S., and Tamada, K., Studies on two-dimensional transonic flows of compressible fluid. Part III, *Quart. appl. Math.* **9,** 2, 129–147, July 1951.

This is Part III of a series of studies in two-dimensional transonic flows [Part I, AMR **3**, Rev. 1982; Part II, AMR **4**, Rev. 2113]. Here a different approximation to the fundamental equations for flow in the hodograph plane is proposed. It is found that if the local supersonic Mach number does not exceed 1.2, the approximation may be expected to yield useful results. By means of this new simplification, the particular integrals of the differential equation reduce to products of Bessel and trigonometric functions.

By a well-known property of Bessel functions, authors are able to construct a solution in closed form which, when transformed back to the physical plane, corresponds to a flow past a 10% symmetric airfoil with a cusped trailing edge. The flow patterns have been calculated for three free-stream Mach numbers: 0.717, 0.745, and 0.752. For 0.717, critical speed is just reached at the minimum pressure point. The second case has a finite local supersonic region after which there follows an extremely large pressure gradient. Due to this high pressure gradient, a limiting line appears at the slightly higher Mach number 0.752. Between the two extreme cases, the boundary shape, however, has changed only very slightly. Y. H. Kuo, USA

3937. Giese, J. H., Stream functions for three-dimensional flows, *J. Math. Phys.* **30,** 1, 31–35, Apr. 1951.

In steady adiabatic three-dimensional flow, the continuity equation defines a solenoidal vector which can be represented as the vector product of two gradients; the streamlines are the intersections of two families of surfaces. Similarly, Crocco's equation defines another solenoidal vector and there are two other families of surfaces intersecting along the streamlines. Thus Stokes' and Crocco's stream functions of plane flow are generalized. Author points out the Gaussian variational principle of Lin can be extended in this case. The variation of entropy in a curved shock wave does not allow the values of the two Crocco's stream functions to be preserved, but this preservation can occur for the two Stokes' stream functions.

F. H. van den Dungen, Belgium

3938. Lukasiewicz, J., Shock tube theory and applications, *Nat. Res. Counc. Canad. mech. Engng. Rep.* MT-10, 108 pp., Jan. 1950.

A shock tube is a constant-pressure tube filled with two (possibly different) gases at different pressures which are separated by a membrane which can be quickly removed. Initially used to calibrate crystal pressure gages and to study unsteady shock reflection, this device has recently been applied to the investigation of steady subsonic or supersonic flows about models, for, although the process after the removal of the membrane is unsteady, there are, at certain cross sections, periods of steady flow.

On the basis of one-dimensional unsteady-flow theory (method of characteristics) the flow is computed and charts for the aerodynamic design of shock tubes are given. Choice of gases in the shock tube, differences between theoretical and experimental flow, pressure level, influence of boundary layer along wall and model, and Reynolds numbers are discussed. On this basis the use of the shock tube as short-time wind tunnel and aeroballistic range is investigated. Further applications are the study of specific heats and of relaxation times in connection with strong shocks. Gottfried Guderley, USA

3939. Wang, C.-T., A note on Bateman's variational principle for compressible fluid flow, *Quart. appl. Math.* **9,** 1, 99–102, Apr. 1951.

This note formulates a variational principle for compressible-fluid-flow problems in an infinite domain. It is essentially a modification of Bateman's variational principle which is valid for finite flow fields. The new principle is derived by adding suitable integrals to the Bateman variational integral. Moreover, the new integral leads to the same Euler equation. References are given to author's work in which this principle has been applied to the approximate solution of compressible-flow problems by means of the Rayleigh-Ritz, Galerkin, and Biezeno-Koch methods.

Henry G. Lew, USA

3940. Spahr, J. R., and Dickey, R. R., Effect of tail surfaces on the base drag of a body of revolution at Mach numbers of 1.5 and 2.0, *Nat. adv. Comm. Aero. tech. Note* **2360,** 25 pp., Apr. 1951.

Recent model tests at high Mach number have shown discrepancies in base pressures. Present results obtained in the 1 ×

3-ft supersonic tunnel at Ames Aeronautical Laboratory show that discrepancy can be explained by the interference effect of the tail surfaces. Physical explanation of this effect is given as well as an approximate analysis. The low pressure field existing over the rear of the tail surface reduces the base pressure and thus increases base drag. This can be eliminated by mounting surfaces at least one chord length ahead of base. By mounting tail surfaces such that they overhang the base, the positive pressure field of the forward part of the surface increases base pressure. No definite conclusion regarding most favorable location can be given due to changes in tail-surfaces drag for various locations. Henry H. Hoadley, USA

Turbulence, Boundary Layer, etc.

(See also Rev. 3933)

3941. Thiriot, K.-H., Boundary-layer flow shortly after a sudden start or stop of a rotating disk (in German), *Z. angew. Math. Mech.* **30**, 390–393, 1950.

In this note, nonstationary laminar boundary-layer flow over a rotating disk is considered for two special cases: (1) A disk in a viscous fluid suddenly starts to rotate with a constant angular velocity ω, and (2) a uniformly rotating disk is suddenly brought to stop. Let (v, u, w) be the velocity components in cylindrical coordinates (r, φ, z), and z the axis of rotation. The boundary conditions for the second problem are: for the time $t \geq 0$ when $z = 0$, $v = u = w = 0$ and $z = \infty$, $u = r\omega$, $v = 0$, and these apply also for the first problem when u is redefined to suppress the rotation of the disk; the form of solutions in both cases is

$$u = \omega r[F_0(\eta) + (\omega t)^2 F_2(\eta) + \cdots]$$
$$v = \omega r[(\omega t)G_1(\eta) + (\omega t)^3 G_3(\eta) + \cdots]$$

with $\eta = z/2(\nu t)^{1/2}$. The functions F_0, F_1, G_1, and G_3 have been integrated and tabulated for both problems. Author mentions that his earlier paper on this same subject [same **Z. 20**, 1–13, 1940] was in error, as pointed out by Görtler [Naturforschung und Medizin in Deutschland 1939–1946, Band 5, pp. 33–73, Dieterich, Weisbaden, 1948]. Y. H. Kuo, USA

3942. Tifford, Arthur N., On the theory of heat transfer through a laminar boundary layer, *J. aero. Sci.* **18**, 4, 283–284, Apr. 1951.

A modification of Lighthill's analysis [AMR **4**, Rev. 2162] of heat transfer through a laminar boundary layer is proposed. The velocity profile is linearized by introducing an "effective u" $= y/\mu[\tau + (4/3)\delta^{**}(dp/dx)]$. The product $(\sigma\tau)$ is then replaced throughout Lighthill's analysis by the new "effective $(\sigma\tau)$." It is shown that this modified Lighthill solution approaches much more closely a correct solution.
F. N. Frenkiel, USA

3943. Cooke, J. C., Pohlhausen's method for three-dimensional laminar boundary layers, *Aero. Quart.* **3**, part 1, 51–60, May 1951.

Wild [AMR **2**, Rev. 510] has extended to three dimensions the von Kármán-Pohlhausen method for the laminar boundary-layer flow over a fixed obstacle. The exact solution being known [by author, AMR **4**, Rev. 764] for the case of an infinite yawed cylinder when the velocity outside the boundary layer over the surface normal to the generators is of the form $U = cx^m$, the Pohlhausen-Wild method is tested in two ways for agreement with the exact solution. In the first way, the method is used without modification. In the second, the method is applied to the cross flow along the generators only, Hartree's exact result [*Proc. Comb. phil. Soc.* **33**, 2, p. 223, 1937] being used for the main flow.

A table of the skin friction, displacement thickness, and momentum thickness is given for various values of $\beta = 2m/(m + 1)$, and the agreement is found to be fairly good for $\beta > 0$ (accelerated flow) but not so good for $\beta < 0$ (retarded flow).
Chia-Shun Yih, USA

3944. Mager, A., Generalization of boundary-layer momentum-integral equations to three-dimensional flows including those of rotating system, *Nat. adv. Comm. Aero. tech. Note* 2310, 47 pp., Mar. 1951.

The Navier-Stokes equations for the steady flow of a fluid of constant density and viscosity are presented in an orthogonal curvilinear coordinate system that rotates with a constant angular velocity about an arbitrary axis in space. The resulting equations are then changed to boundary-layer equations by making the usual assumptions of boundary-layer theory; the equations are given both for laminar and for turbulent flow. An integration of the boundary-layer equations results in two general boundary-layer momentum equations which are partial differential equations. By imposing restrictions on the velocity profiles of the velocities along and normal to the direction of a streamline at the outer edge of the boundary layer, two partial differential equations are obtained, one for the momentum thickness in the direction along the outer streamline, the other for the boundary-layer deflection angle at the surface. The equations are solved and the results compared with experiment for the special case of flow in a curved nonrotating duct. Agreement between theory and experiment is good. Neal Tetervin, USA

3945. Ross, D., and Robertson, J. M., A superposition analysis of the turbulent boundary layer in an adverse pressure gradient, *J. appl. Mech.* **18**, 1, 95–100, Mar. 1951.

In order to calculate the velocity profile in an adverse pressure gradient, especially in convergent and divergent channels, the velocity u is subdivided into two components: u_{friction}, which can be calculated by the Prandtl-von Kármán logarithmic relation, and u_{pressure} for which an empiric law derived from measurements in convergent and divergent channels by Nikuradse holds. Author stresses that the obtained velocity profile represents an interim solution which yields satisfying results for the velocity itself, but not for its derivatives, with the exception of the wall shear stress. Manfred Schaefer, Germany

3946. Granville, P. S., A method for the calculation of the turbulent boundary layer in a pressure gradient, *David W. Taylor Mod. Basin Rep.* 752, 40 pp., May 1951.

A method is described for calculating the development of incompressible turbulent boundary layers in pressure gradients, so that drag and separation of flow may be predicted more accurately. The principal contribution is the formulation of new relations for the variation with pressure gradient of (1) the shearing stress at wall (local skin friction), and (2) integral of the shearing stresses across the boundary layer for use in a moment-of-momentum equation recently derived by Tetervin and Lin.

With the assumption of the one-parameter characterization of velocity profiles by a shape parameter H, the moment-of-momentum equation and the von Kármán momentum equation constitute a working method involving the solution of a pair of simultaneous first-order differential equations.

As an introduction to the subject of turbulent boundary layers in pressure gradients, a review of previous methods is included.
From author's summary by Morris W. Rubesin, USA

3947. Batchelor, G. K., Pressure fluctuations in isotropic turbulence, *Proc. Camb. phil. Soc.* **47,** part 2, 359–374, Apr. 1951.

Paper considers statistically the pressure field in a turbulent fluid. Correlation values of the pressure at two different points are derived in terms of a fourth-order moment of the turbulence velocity. The mean-square pressure fluctuation has the value $p^2 = 0.34\rho^2(v^2)^2$. The mean-square pressure gradient has been expressed in terms of the mean square of the velocity and a length parameter.

Author arrives at same results as Heisenberg and Obukhov for the correlation of the pressure fluctuations. Experimental results of R. W. Stewart are included to show the ratio of the fourth moments to the square of the second moments at different distances downstream from a grid. The hypothesis of normal distribution of velocities is examined. Reviewer believes that correctness of latter assumption of normality is questionable, although it does not affect excellent treatment of pressure fluctuations. Louis M. Laushey, USA

3948. Stewart, R. W., and Townsend, A. A., Similarity and self-preservation in isotropic turbulence, *Philos. Trans. roy. Soc. Lond. Ser. A,* **243,** 867, 359–386, June 1951.

Authors discuss extent to which statistical properties of isotropic turbulence are independent of the sources of turbulence based on experimental measurements of double- and triple-velocity correlation functions and energy spectrum. Local similarity considered by Kolmogoroff is valid only for wave numbers k larger than $0.6\ (\epsilon/\nu^3)^{1/4}$, where ϵ is the total energy description per unit mass and ν is the kinematic viscosity. The inertial range in which the energy spectral density varies as $k^{-5/3}$ appears only for Reynolds numbers of the turbulence based on the microscale λ greater than 1730, which are far greater than those found in laboratory measurements. It is concluded that all suggested forms for the inertial transfer term in the spectrum equation are in error. Within the initial period of decay, the greater part of energy spectrum function is self-preserving, and this part has a shape independent of the shape of a turbulence-producing grid. Hugh L. Dryden, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 3752, 3924, 3934, 3935, 3968)

3949. Rościszewski, J., Longitudinal dynamic stability of an aircraft in high speed flight (subsonic) (in Polish), *Techn. Lotn.* **6,** 1, 13–23, Mar. 1951.

Paper reviews basic equations for study of longitudinal dynamic stability of an airplane. Particular attention is given to the effects of compressibility at high subsonic speeds. The characteristic irregularities arising at Mach numbers above 0.7 are illustrated by examples of several types of airplanes.
M. J. Thompson, USA

3950. Frager, L., Measurements of the stability of aerodynes (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 323–333, 1950.

Note gives very general description of method of determining, by means of forced oscillations of low frequency applied to an airplane model in a wind tunnel, the following characteristics: (1) Steady aerodynamic derivatives; (2) characteristic equation; and (3) response functions. For the airplane itself, response functions can be measured in flight and the characteristic equation is calculated from one of them. No information about experimental approach is given and no results are presented.
A. I. van der Vooren, Holland

3951. Bihrle, W., Jr., Floating characteristics of a plain and a horn-balanced rudder at spinning attitudes as determined from rotary tests on a model of a typical low-wing personal-owner airplane, *Nat. adv. Comm. Aero. tech. Note* 2359, 89 pp., May 1951.

Results of rotary tests at spinning altitudes to determine the floating characteristics of a plain and a horn-balanced rudder on a model of a low-wing personal-owner airplane are presented. At the higher spinning angles of attack, either rudder oscillated violently as a result of being in the wake of the stalled wing. At the lower spinning angles of attack or in the absence of the wing, the horn-balanced rudder had more favorable floating characteristics than the plain rudder, but neither rudder would accomplish spin recovery with the rudder free. It is shown that, as a result of interference from the horizontal tail, an appreciable difference existed between rudder-floating angles obtained from static and from rotary tests. Robert M. Crane, USA

3952. Lomax, H., Heaslet, M. A., and Sluder, L., The indicial lift and pitching moment for a sinking or pitching two-dimensional wing flying at subsonic or supersonic speeds, *Nat. adv. Comm. Aero. tech. Note* 2403, 56 pp., July 1951.

The indicial lift and moment response to a unit step increment of either angle of attack or pitch angle are obtained for a two-dimensional flat plate in linearized compressible flow at $M = 0.8$, 1.2, and 2. The solutions are obtained by direct analogy with steady-state linearized three-dimensional lifting-surface theory by replacing the spanwise coordinate with the time variable. Paper is an extension of authors' previous work [AMR **2,** Rev 1420].

To illustrate the method, Wagner's classic integral equation, representing the indicial response due to a unit step in the angle of attack of a flat plate in two-dimensional incompressible flow, is rederived by direct application of the steady three-dimensional linearized thin-wing theory for slender planforms.
E. V. Laitone, USA

3953. Holme, O. A. M., Measurements of the pressure distribution on rectangular wings of different aspect ratios, *Flygtekn. Forsöksanst. Medd.* **37,** 27 pp., 1950.

Author describes experiments on a symmetrical airfoil of 9.2% thickness ratio. The observed pressure distributions, section and wing normal-force coefficients, and section and wing centers of pressure positions for various angles of attack are given. The effect on these aerodynamic properties of varying the aspect ratio from 0.52 to 4.50 is illustrated. Results are compared with theoretical predictions. Morris S. Macovsky, USA

3954. Miele, A., Problems of minimum time in the unsteady flight of aircraft (in Italian), *Atti. Accad. Sci. Torino* **85,** 41–52, 1950–1951.

Determination of distribution of velocity V with altitude z leading from fixed initial conditions to given final flight conditions in minimum time is discussed. Airplane weight is considered constant and thrust T is supposed to depend on speed and altitude; flight conditions where thrust is greater than aerodynamic drag R are considered.

After having formulated the problem in a general way by the calculus of variations, author argues that solutions of practical interest can be obtained if squares of path inclination and centrifugal forces are neglected. Difficulties arising from variational methods are overcome by a proper transformation using Gauss-Green theorem; the devised procedure allows a clear, unified discussion of optimum flight techniques for both absolute or conditioned minima. Best speed distributions $V = V(z)$ have two end branches depending on boundary conditions and

a common central part defined by $g(\partial\psi/\partial V) = \partial(\psi V)/\partial z$ (where $\psi = 1/(T - R)V$).

Paths of minimum fuel consumption are analyzed and similar results achieved by the same method. All the solutions show a common part defined by $g\partial(cT\psi)/\partial V = \partial(cTV\psi)/\partial z$ (where c is specific fuel consumption).

The basic hypotheses are discussed and numerical applications to a typical turbojet fighter carried out, showing a comparison with the known solution neglecting influence of tangential inertia forces, on optimum speed. P. Cicala, Argentina

3955. Harmon, S. M., Method for calculating downwash field due to lifting surfaces at subsonic and supersonic speeds, *Nat. adv. Comm. Aero. tech. Note* 2344, 30 pp., Apr. 1951.

A method is developed by which the downwash behind a wing at supersonic and subsonic speeds can be determined, utilizing sources which are distributed over the planform of the wing in a manner determined from the loading on the surface. Method is applied to the downwash field of uniformly loaded wings, in which case the source distribution reduces to line sources along the edges. Generalized formulas are given for swept and rectangular wings. It is also indicated how to obtain the downwash due to wings of arbitrary loading. Examples are given in which specific formulas are derived for a rectangular wing at supersonic speeds for a uniform loading and for a linear chordwise variation of loading. Tore Gullstrand, Sweden

3956. van Heemert, A., The calculation of the downwash at the surface of a lifting plane in steady flow. I. Theory. II. Tables and graphs, *Nat. LuchtLab. Amsterdam Rap.* F.58, 40 pp., 16 pp., Mar. 1950.

Extensive summary of two previous reports by the same author in which a method of calculating the downwash at the wing surface of a lifting plane in steady incompressible flow was developed. Method is applicable to wings with sufficiently regular planforms, including sweptback wings, but discontinuities in tangents to the leading or trailing edges are excluded. A complete set of tables and graphs as well as examples of their application are also given. Harvard Lomax, USA

3957. Diederich, F. W., Charts and tables for use in calculations of downwash of wings of arbitrary plan form, *Nat. adv. Comm. Aero. tech. Note* 2353, 37 pp., May 1951.

A series of tables and charts give values of a dimensionless function proportional to downwash induced by a single, rectangular horseshoe vortex in incompressible fluid. Method is presented for representing spanwise load distribution on wings with wide range of planform, taper, sweep, and aspect ratio using a system of 21 such vortexes. Graphical or tabular procedure then gives downwash angle at tail location for stability calculation, or induced velocities at a network of wing points for load-distribution computation by the Falkner method. Subsonic compressibility correction can be made readily by Prandtl-Glauert transformation.

Four-decimal-place accuracy and small tabular interval appear adequate for most engineering purposes. Refined table is provided for points close ahead and behind lifting segment of vortex, where downwash varies rapidly with longitudinal distance. Supplementing data for plane of the vortex, tables are given for planes 1, 2, 4, and 6 vortex spans above and below that plane.

Comparison between experimental and predicted values of downwash behind a 30° sweptback wing of aspect ratio 4.5 indicates reasonable confirmation except at points close to trailing edge. Although author's suggestion that his method is inapplicable for high angle of attack, low aspect ratio, large sweep, etc., is certainly sound, reviewer believes general simplicity and applicability of these charts and tables render them a very valuable tool for many aeronautical calculations. Holt Ashley, USA

3958. Piland, R. O., Some theoretical characteristics of trapezoidal wings in supersonic flow and a comparison of several wing-flap combinations, *Nat. adv. Comm. Aero. tech. Note* 2336, 26 pp., Apr. 1951.

Lift and pitching moment due to angle of attack and pitching, and lateral force and yawing moment due to rolling are derived for a trapezoidal wing on basis of usual linearized theory. Variation of derivatives with governing parameters are shown in chart form. Some of these expressions were derived previously by others [cf. AMR **1**, Rev. 499; AMR **3**, Rev. 2403]. Evvard's method is used to determine velocity potential, and stability derivatives are obtained by integrating forces and moments over the wing. Validity of method is restricted to cases when inboard Mach line from a leading-edge tip crosses trailing-edge, and side edge is subsonic.

Useful comparison is made of lift due to flap deflection and the rolling effectiveness for half delta and tip (point forward) and trailing-edge flaps on rectangular and triangular wings. In general, triangular wings show most effectiveness with either type flap. J. S. Isenberg, USA

3959. Gabrielli, G., A suggestion for the determination of taper ratio of monoplane wings (in Italian), *Termotecnica* **5**, 2, 67–71, Feb. 1951.

Though the taper ratio of a wing is important for aeroelastic studies, for structural weight analyses, and for stall problems, no precise definition of it exists.

Author suggests a new mathematical definition of the taper ratio of wings, which is applicable to any arbitrary planform. He replaces each planform by a member of the wing family $2c(y)/b = 2c_0/b(2y/b)^{1/n}$, where $c(y)$ is the local wing chord, b the wing span, y the coordinate in span direction, c_0 the chord in the plane of symmetry, and the exponent n a constant which is related to the aspect ratio A by $A = b/c_0(1 + 1/n)$. For a triangle, n equals 1, and for a rectangular wing, n equals ∞. The taper ratio r is defined by $r = (b^2/S)^{1/n}$. For a triangular wing this gives not zero, but 2.4. Gerhard W. Braun, USA

3960. Kline, D. B., and Walker, J. A., Meteorological analysis of icing conditions encountered in low-altitude stratiform clouds, *Nat. adv. Comm. Aero. tech. Note* 2306, 27 pp., Mar. 1951.

Authors present data on liquid-water content, droplet size, and temperature measured during 22 winter flights (1948–1949 and 1949–1950) in stratiform clouds. Analysis of meteorological conditions observed during 74 flights over four winters indicated inverse relationship of liquid-water concentration to maximum horizontal extent of icing clouds. Icing conditions occurred most frequently in the southwest and northwest quadrants of low-pressure areas. Icing conditions were usually encountered at subfreezing temperatures in nonprecipitating stratiform clouds, icing being unlikely in areas where most surface stations were reporting precipitation. Measurements of liquid-water content obtained during 12 flights near the time and location of radiosonde observations were compared with theoretical values. Average liquid-water content of a (stratiform) cloud layer, as measured by the "multicylinder" technique, seldom exceeded two thirds of that which could be released by adiabatic lifting, as determined from radiosonde analysis. Authors describe apparatus and methods of measurement, and indicate that sufficient data are now at hand to provide tentative design criteria for thermal anti-icing systems K. H. Jehn, USA

3961. Young, A. D., and Hufton, P. A., Note on the lift and profile drag effects of split and slotted flaps, *Aero. Counc. Lond. Rep. Mem.* 2545, 20 pp., Sept. 1941, published 1951.

From results of a careful survey of available data on lift and profile drag increments of flaps, authors develop a method for prediction of lift and profile drag increments for full and part-span split and slotted flaps. Method is based on assumption that increments can be expressed as products of a function of ratio of flap chord to extended chord by a function of flap angle. Assumption has some theoretical justification for lift, but none for profile drag. Authors conclude from experimental results examined that method is satisfactory to predict lift increments, but that care must be exercised in estimating profile drag increments since method is, in this case, empirical and available data are not too reliable. Maurice André Brull, USA

3962. Moffat, R. L., and Gaukroger, D. R., Air bags as flexible supports in ground resonance testing of aircraft, *Aero. Res. Counc. Lond. curr. Pap.* 32, 6 pp., Sept. 1949, published 1950.

Low natural-frequency load supports are desired in studying aircraft resonance. A pressurized bag may be designed for low frequency response in accordance with an improved formula considering bag-pressure differential from atmosphere. Good agreement with measured bag frequency is obtained. Frequency is shown to decrease as pressure increases, but structural limitations (above 2 psi differential) prevent reduction to desired value of 1 cps. Reduction to 2.1 cps is obtained and might be made to ≈ 1.3 cps by following the principles outlined.

H. M. Spivack, USA

3963. Michael, W. H., Jr., Analysis of the effects of wing interference on the tail contributions to the rolling derivatives, *Nat. adv. Comm. Aero. tech Note* 2332, 37 pp., Apr. 1951.

The angularity of the airstream at the vertical tail and the resulting forces and moments due to rolling are calculated for various airplane configurations. Wing aspect ratio and sweepback, vertical tail span, and factors of angle of attack and airplane geometry appear important. Sidewash is calculated for a limited range of wing planforms and vertical tail sizes. Comparison with experimental data show fair agreement, especially when the wing-interference effects are included.

H. P. Liepman, USA

3964. Siefert, G., A third order boundary value problem arising in aeroelastic wing theory, *Quart. appl. Math.* **9**, 2, 210–218, July, 1951.

The differential equation of the type $d^2/dx^2EI(x)dz/dx - \lambda c(x)z = 0$ with the boundary conditions

$$z(0) = z'(l) = (EI(x)z'(x))'_{x=l} = 0, \cdots \quad [1]$$

represents a nonself-adjoint boundary-value problem. Part I of this paper deals with the characteristic values of the system

$$u'''(x) + p(x)u'(x) + [q(x) + \lambda]u(x) = 0;$$
$$u(0) = u'(0) = u''(1) = 0 \quad [2]$$

where p and q are real-valued functions analytic on $0 \leq x \leq 1$. Author finds that this system has an infinite number of real characteristic values and that all characteristic values are real if the upper bounds of $|p(t)|$ and $|q(t)|$ and the positive numbers $|p(1)|$, $\int_0^1 |p(t)|dt$, $\int_0^1 |r(t)|dt$, where $r(t) = q(t) - p'(t)$, are small enough. Part II indicates a pair of transformations which takes the system

$$[f(t)y''(t)]' + \lambda g(t)y(t) = 0;$$
$$y(0) = y'(0) = y''(1) = 0 \quad [3]$$

where $f(t) > 0$, $g(t) > 0$, are real and analytic on $0 \leq t \leq 1$, over into a system of the form [2].

The adjoint of [3] is of the form [1], and consequently the characteristic values of [3], being identical with those of its adjoint, are of interest in aeroelastic wing theory.

From author's summary by H. G. Loos, Holland

Aeroelasticity (Flutter, Divergence, etc.)

3965. Dörr, J., Determination by calculation and by wind-tunnel tests of the critical velocities of a plane model with two degrees of freedom: Rotation of the wing and rotation of the aileron (in French), *Off. nat. Étud. Rech. aéro. Note tech.* 3, 12 pp., Mar. 1951.

Report compares the critical flutter speeds of a wing-aileron system, in which the aileron has a setback hinge. There is good agreement with theoretical results neglecting the setback hinge, the theoretical aerodynamic forces being multiplied by a factor 0.7. A. W. Babister, England

3966. Weber, R., and Ruppel, W., Flutter study at supersonic speeds. Flexural-torsional flutter (in French), *Off. nat. Étud. Rech. aéro. Note tech.* 2, 15 pp., Feb. 1951.

Flutter calculations are presented for the two-dimensional supersonic case of a wing with two degrees of freedom, translation (representing bending), and rotation about a spanwise axis (representing torsion). Methods applied are conventional. Aerodynamic coefficients have been borrowed from existing theories (which are still unchecked experimentally). Results comply in many respects with well-known subsonic trends. The position of the torsional axis and the natural frequency in torsion are of predominant importance. J. H. Greidanus, Holland

3967. Gossard, M. L., An iterative transformation procedure for numerical solution of flutter and similar characteristic-value problems, *Nat. adv. Comm. Aero. tech. Note* 2346, 75 pp., May 1951.

The iterative transformation procedure suggested by H. Wielandt for numerical solution of flutter and similar characteristic value problems is described. Whereas the original and translated works of Wielandt are difficult to follow, author explains clearly the idea of the iterative transformation procedure and applies it to a vibration problem and a flexure-torsion flutter problem. The principle of the iterative transformation procedure is similar to the usual iteration process for solving characteristic value problems. In the ordinary vibration problem, both procedures are identical for finding the first mode. The distinguishing features of the iterative transformation procedure occur in the determination of solutions higher than the fundamental mode. The method does not employ the usual orthogonality relations in its sweeping operations, but makes use of "forcing functions" which apparently reduce the labor and increase the accuracy. The flutter problem is solved by the same method as used in the ordinary vibration problem. This is possible if it is formulated in terms of a pseudoflutter problem in which the problem is altered by introducing an artificial damping so that the complex eigenvalue is given by $\omega^2/(1 + ig_a)$ where ω is the flutter frequency and g_a is a coefficient of artificial damping. This makes it possible to assume real values of k and obtain real eigenvalues. True flutter is, of course, possible only for those special cases in which $g_a = 0$. The fact that most of the functions involved are complex increases the labor in the flutter problem to about four times that required in the ordinary coupled natural-vibration problem. R. L. Bisplinghoff, USA

3968. Babister, A. W., Flutter and design of sweptback and sweptforward wings, *Coll. Aero. Cranfield Rep.* 39, 13 pp., June 1950.

Report extends the method of treating straight and tapered cantilever wings used in two earlier papers [*Aero. Res. Counc. Rep. Mem.* 1782, 1869] to the case of the swept wing (back or forward). Assuming wing to be semirigid and fixed at the root, and taking the modes of displacement to be linear in torsion and parabolic in flexure, equations of flexural-torsional flutter are derived. The critical speeds for flutter and wing divergence are determined for incompressible flow and, by using a modified Glauert correction, for compressible flow.

In applying the results to a wing of aspect ratio 5 and taper ratio $1/2$, author finds that minimum flutter speed occurs for sweepback angles of 5° to 20°. For highly sweptback or forward wings, the flutter speed is double that for unswept wings with the same wing stiffness. He also finds that, for a sweptforward wing, divergence will occur before flutter. The effects of change of flexural or torsional stiffness and position of inertia axis are investigated. Ernest W. Anderson, USA

3969. Buxton, G. H. L., and Sharpe, G. D.; Scruton, C., and Dunsdon, Miss D. V., Part I, Ternary tailplane—elevator—tab flutter. Part II. Experiments on the influence of tab mass-balance on flutter, *Aero. Res. Counc. Lond. Rep. Mem.* 2418, 18 pp., Nov. 1946, published 1951.

Part I presents the results of theoretical analyses showing that flutter in the ternary mode is possible even though all binary modes are stable. Of practical interest is the conclusion that mass overbalance of tabs is not a cure-all for tab flutter and may even be unfavorable as compared to 100% static balance. Part II contains the results of wind-tunnel tests of a flutter model which confirm the conclusions of part I. Benjamin Smilg, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 3822, 3922, 3925, 3994)

3970. Rogallo, V. L., Roberts, J. C., and Oldaker, M. R., Vibratory stresses in propellers operating in the flow field of a wing-nacelle-fuselage combination, *Nat. adv. Comm. Aero. tech. Note* 2308, 33 pp., Mar. 1951.

Conventional three- and four-bladed, hollow, steel propellers of about 13-ft diam were tested in the flow field of a wing-nacelle-fuselage combination. Wake surveys measured the thrust to give the magnitude of the oscillating airloads on the propeller, while strain gages measured the first-order vibratory stresses.

Propeller was operated at tunnel speeds up to 173 mph, rotational speeds up to 1265 rpm, blade angles 12.5° and 20° at $0.75R$, and angles of attack from $-2°$ to $+10°$. Conclusions reached are that the steady-state propeller theory is adequate for prediction of magnitude and distribution of oscillating air loads, provided the complete flow-field characteristics are used; with these air loads, the first-order, nonresonant, vibratory stresses can be calculated accurately. H. P. Liepman, USA

3971. Stack, J., Draley, E. C., Delano, J. B., and Feldman, L., Investigation of the NACA 4-(3)(08)-03 and NACA 4-(3)(08)-045 two-blade propellers at forward Mach numbers to 0.725 to determine the effects of compressibility and solidity on performance, *Nat. adv. Comm. Aero. Rep.* 999, 32 pp., 1950.

Systematic tests of two 2-blade propellers having NACA 4-(3)(08)-03 and NACA 4-(3)(08)-045 blade designs are described in detail in order to establish the changes in propeller characteristics due to compressibility and solidity. The tests were conducted at 10 different Mach numbers ranging from M = 0.165 to 0.725.

It is clearly established that: (1) Serious losses in propeller efficiency occurred at tip Mach numbers in excess of 0.91; (2) the range of peak efficiencies decreased markedly with increase of forward speed; (3) compressibility losses could be delayed to a higher forward Mach number by decreasing the tip Mach number through operation at increasing values of blade angle; (4) losses in propeller efficiency due to compressibility effects decreased with increase of blade width; (5) the range of power-disk loading for high efficiency decreased with increase of forward speed. This decrease in range of high efficiency indicates that propeller designs for high-speed craft may be critical and may necessitate the design of propellers to fit specific applications.

Test data indicate that efficiencies somewhat greater than 90% can be obtained at $M = 0.7$, provided that the advance ratio and solidity are maintained in specific ranges. Consequently it appears probable that currently obtained low-speed efficiencies can be maintained to sea-level speeds of the order of 550 mph. The report contains material originally issued in 1944 as NACA ACR 4A10 and ACR 4B16. Leonard Goland, USA

3972. Burns, J. C., Airscrews at supersonic forward speeds, *Aero. Quart.* **3,** part 1, 23–50, May 1951.

A method is given for determining the performance of a windmill in a uniform supersonic stream. Linearized equations are used and it is assumed that no interference effects between blades are present. Thus, the results obtained are valid only for conditions in which the rotational component of tip speed is small by comparison to sonic speed. Reviewer notes that the operating range of best efficiency for both windmills and propellers requires rotational tip speeds of the same order of magnitude as the forward speed, and that this important range apparently cannot be treated by the method given in this paper.

John V. Becker, USA

3973. Simonis, E. A., and Reeman, J., Gas turbine design based on free vortex flow, *Aero. Res. Counc. Lond. Rep. Mem.* 2541, 23 pp., May 1944, published 1951.

Design of single-stage and two-stage turbines based on free-vortex flow through nozzles and blades is discussed. Limiting factors in turbine design, including disk stress, centrifugal blade stress, entrance Mach number, gas-bending stress in blading, blade-height ratio, and tip-to-root radius ratio are considered.

Calculations of turbine-stage performance are presented in chart form for various values of nozzle angle, of degree of reaction in the nozzles and blades, and of swirl in the nozzles and blades. These calculations are based on flow conditions at the root of the blades and not on those at the mean diameter, since the former are of greater importance in limiting the turbine design. The many charts presented are useful to the designer in obtaining rapid estimates of a turbine stage to meet given specifications of flow and power. An example illustrating the use of these design charts is given. Joseph Kaye, USA

3974. Alpert, S., and Litrenta, R. M., Construction and use of charts in design studies of gas turbines, *Nat. adv. Comm. Aero. tech. Note* 2402, 38 pp., 11 figs., July 1951.

A method based on certain simplifying assumptions is presented for the computation and graphic representation of a series of possible turbine designs for any specific application. Paper shows the effects of both stress and flow limitations on turbine design. A preliminary design chart is used for determining the number of stages and the effects of annular area divergence and exit whirl velocity. A specific design chart is recommended for

analysis of turbine diameter, Mach numbers, turning angles, and blade stresses. A large chart for designers is enclosed, and examples for using design charts are presented.

H. E. Sheets, USA

3975. Hall, E. W., and Wilcox, E. C., Theoretical comparison of several methods of thrust augmentation for turbojet engines, *Nat. adv. Comm. Aero. Rep.* 992, 11 pp., 1950.

This is an excellent analytical investigation using assumptions consistent with current practice. All results are presented in graphical form. The investigation covers (1) tail-pipe burning; (2) water injection at the compressor inlet; (3) combination tail-pipe burning plus water injection; (4) bleedoff; (5) rocket-assist methods of thrust augmentation for turbojet engines. Effect of augmented liquid ratio on augmented thrust ratio and the effects of altitude and flight Mach number on the performance of the various methods are determined. The additional take-off weight involved by use of the different thrust-augmentation methods is also investigated, as well as the effect of the various thrust-augmentation methods on the range of a representative aircraft.

Results indicate that the combination tail-pipe burning plus water-injection method is best for large amounts of thrust augmentation and that the tail-pipe burning method is best for smaller amounts, inasmuch as both methods have lower augmented liquid ratios for given augmented thrust ratios than any of the other methods considered.

For take-off conditions, the maximum augmented thrust ratio for the combination tail-pipe burning plus water-injection method is 1.9 at an augmented liquid ratio of 7. For the tail-pipe burning method, maximum augmented thrust ratio is 1.5 at an augmented liquid ratio of 4. An increase in flight Mach number greatly increases the augmented thrust ratio produced by all methods investigated. Increasing altitude decreases the augmented thrust ratio somewhat for those methods employing water injection and has a very small effect on augmented thrust ratio for the tail-pipe burning method. Increasing the engine-compressor pressure ratio increases the maximum attainable augmented liquid ratio and thereby increases the maximum possible augmented-thrust ratio.

A comparison on the basis of additional take-off weight indicates that the best method of augmentation depends on the required amount of thrust augmentation and that each method was best for a certain range of augmented-thrust ratios. For a representative aircraft operating at a flight Mach number of 1.50 and an altitude of 35,332 ft, the tail-pipe burning method allowed a slight increase in maximum range and a considerable increase in disposable load. The other methods allowed considerable increase in disposable load at the expense of reduced range.

From authors' summary by Ammon S. Andes, USA

Flow and Flight Test Techniques

(See also Revs. 3909, 3933, 3938, 3948, 3953, 3969)

3976. Rhoden, H. G., The development of a wind tunnel, *Engineering* **171,** 4454, 677–680, June 1951.

Design of a low-speed wind tunnel and means for improving the flow distribution are discussed. The tunnel is described as a low turbulence, large flow-volume supply for testing compressor blade cascades. It is powered by a 150-hp motor and centrifugal blower on the pressure side of an open-circuit arrangement. The development has gone to some length in the use of honeycombs, perforated plates, and screens to smooth the flow and obtain some reduction in the turbulence level. H. M. Spivack, USA

3977. Lamplough, M. A., Shock-wave shadow photography in tunnel and in flight, *Aircr. Engng.* **23,** 266, 94–103, Apr. 1951.

A method for three-dimensional shadowgraphs is explained and demonstrated with the aid of a transparent model. Some methods of reconstructing shock waves for a sweptback wing from original recorder photographs are discussed. In part II this method, together with some variants and special arrangements, is applied to shock-wave photography in free flight. Different difficulties are summarized and suggestions are made on how to overcome them. The possibility of determining the position of shock waves is discussed briefly. Part III describes the shadow characteristics of concave and convex shock waves and the effect of optical systems as they are used in photography of wave profiles during flight.

The appendix deals with the application of separated slits and cylindrical lenses in order to overcome difficulties of unwanted influence of daylight. S. F. Erdmann, Sweden

3978. Gracey, W., and Scheithauer, E. F., Flight investigation of the variation of static-pressure error of a static-pressure tube with distance ahead of a wing and a fuselage, *Nat. adv. Comm. Aero. tech. Note* 2311, 26 pp., Mar. 1951.

Pressure tubes were located $^1/_4$ to 2 chords ahead of the wing tip of a trainer airplane, $^1/_2$ to $1^1/_2$ body diam ahead of the fuselage, and 1 chord ahead of the wing tip of a fighter airplane. Calibrations were obtained over an indicated speed range from stall to 265 mph. In the range of low-lift coefficient, the errors decrease progressively as the distance ahead of the wing is increased, while near maximum-lift coefficient the errors are approximately the same for all positions. In the fuselage-nose installations, the errors are reduced with increasing distance throughout the entire range of the lift coefficient. A comparison of the two types of mounting shows that the change in static pressure error over the lift-coefficient range was of the same order for the $1^1/_2$-diam fuselage-nose installation and for a 1-chord wing-tip installation on the same airplane. George Rudinger, USA

3979. Weltmann, R. N., Fairweather S., and Papke, D., Application of X-ray absorption to measurement of small air-density gradients, *Nat. adv. Comm. Aero. tech. Note* 2406, 41 pp., July 1951.

Authors use either Geiger-Mueller counters or photographic films to measure x-ray intensities. The tungsten target x-ray tube is operated at a few kilovolt and precautions are taken to reduce absorption outside the test section to a minimum. Counter measurements are corrected for the effects of finite resolving time. Two counters are used, one to measure x-ray intensity after passage through test section and the other to correct for fluctuations of the primary beam. A device is described briefly that records the logarithm of the time elapsed for a predetermined number of counts which is a direct function of air density. With a 0.002-in. receiving slit, method was successfully applied to evaluation of a flat-plate boundary layer.

The photographic method has the advantage that no scanning is required and a single exposure yields entire density distribution. Effect of beam spreading in the test section is discussed and a corrective procedure suggested. Although film blackening was apparently well reproducible, it is not clear whether this was established by not developing test films simultaneously. Reviewer believes that in order to obtain reliable results, the characteristic curve of the film (which depends both on the film and on the developing process) should not be assumed to be constant but should be determined in every case. The method was, however, successfully used for measuring gas density in a test cell.

The sensitivity of either method depends on a number of con-

ditions, and sample calculations are presented to determine optimum operation. George Rudinger, USA

3980. Little, B. H., Jr., and Wilbur, S. W., Turbulence-intensity measurements in a jet of air issuing from a long tube, *Nat. adv. Comm. Aero. tech. Note* 2361, 30 pp., May 1951.

Hot-wire measurements were made of the fractional longitudinal turbulence intensity u'/u, and radial intensity v'/u, in a 2-in. jet issuing from a pipe. This jet was bounded by an annular jet having a maximum velocity equal to the mean velocity in the inner pipe.

At the pipe exit, turbulence is not isotropic, u'/u ranging from 2.5 times v'/u in the center to 3 or 4 times near the edge of the jet. u'/u generally increases with Re and x/D (measurements to $5\ x/D$). v'/u ranges from about 0.006 at the center to about 0.013 near the edge, and is generally independent of Re and x/D. Robert S. Levine, USA

3981. Fleming, F. W., and Binder, R. C., Study of linear-resistance flowmeters, *Trans. Amer. Soc. mech. Engrs.* **73,** 5, 621–624, July 1951.

Authors discuss the general characteristics of linear flowmeters of the porous plug type. Experimental data is presented showing regions of departure from linearity. Plugs of various lengths and specific weights are investigated. Authors suggest various dimensionless ratios for describing flowmeter characteristics.

J. F. Manildi, USA

Thermodynamics

3982. Hall, N. A., Thermodynamics of fluid flow, New York, Prentice-Hall, Inc., June 1951, x + 278 pp. $5.50.

This engineering text deals with one-dimensional steady flow of enclosed fluids. Main emphasis is on theory and especially that theory pertaining to perfect gases. Caliber of material presented is advanced undergraduate to graduate. The context by chapter headings: 1. Introduction; 2. Fluid friction; 3. Energy and continuity; 4. Momentum principles; 5. Incompressible fluids; 6. Perfect gases; 7. Constant-area adiabatic flow; 8. Varying-area adiabatic flow; 9. Shock regions; 10. Flow measurement and control; 11. Diabatic flow; 12. Propulsion systems. The appendix contains tables of derived flow functions. Problems are provided for each chapter.

Figures are used liberally to illustrate the nature of empirical coefficients and derived flow functions. The text is noteworthy for inclusion of numerous intermediate steps and associated reasoning in the derivations. This feature tends to make the text self-sufficient and should increase its value to nonstudent readers. A. D. St. John, USA

3983. Nesselmann, K., Fundamentals of applied thermodynamics [Die Grundlagen der angewandten thermodynamik], Berlin, Springer-Verlag, 1950, xi + 320 pp. DM 18.

Book surveys those topics of thermodynamics which are of immediate concern to the mechanical and chemical engineer. Presentation follows the pattern familiar from books by Schüle, Bosnjakovic, and Gröber-Erk; however, the variety of thermodynamic phenomena included here is so large that in many cases (e.g., binary systems) only a listing rather than a thermodynamic treatment has been devoted to them. As a first introduction to applied thermodynamics, the book will give the reader an idea of the variety of problems occurring in engineering practice and of methods available for their solution, and will guide him to the study of more detailed texts, some of which are quoted by author. Its weaknesses are evident where the presentation of fundamental laws is attempted. Two examples: In the discussion of entropy, the Clausius integral is omitted, and since the author (and not only *this* author) interprets dQ as heat supplied from outside only, he is led to observe that $ds = dQ/T$ (Eq. 121) is not valid for a throttling process; or, the thermodynamic potential is introduced only in Eq. 613, rather than being used as the starting point in the derivation of the various equilibrium laws, such as the Clausius-Clapeyron equation.

The wealth of topics which have been treated briefly include: perfect gases and application to power cycles, Maxwell relations, vapors and van der Waals equation, refrigeration cycles, flow through nozzles, liquefaction of gases, gas mixtures, binary systems with distillation and rectification (very rudimentary), chemical reactions (with combustion), Nernst's law. A sixty-page section on the fundamentals of heat conduction, convection, and radiation as well as diffusion is added. Numerous completely solved examples illustrate some chapters of the text.

Eric F. Lype, USA

3984. Bechert, K., Theory of ignition limits of combustible gas mixtures (in German), *Ann. Phys.* (6), **7,** 3–4, 113–128, Apr. 1950.

Author attempts to deal theoretically with the question of rich and lean ignition limits, including effects of pressure, temperature, and fuel type. Limiting mixture composition for ignition will be a function of r_0, radius of a small zone of burned gas at burned-gas temperature and pressure. Ignition of whole mixture is thus a matter of flame spreading. Approach of author is to relate critical r_0 dimension to normal flame thickness Δx; it is expected to be somewhere in the range $0.1\ \Delta x$ to Δx. In previous work, author developed theoretical expression for Δx in terms of combustible gas parameters. Thus, one obtains relation of rich and lean limits of ignition to r_0, initial pressure and temperature, mixture ratio fuel type, and other parameters.

Author's theory in this and previous papers leads to conclusion that burning velocities at rich and lean limits will be alike, and that maximum burning velocity will occur on rich side of stoichiometric.

Extensive discussion is given also of spark ignition. Attention is centered on spark voltage, length, and cross section. Reviewer believes that greater attention should be given to ignition energy.

Several comparisons between theory and experiment are given with agreement ranging from fair to discrepancies by factor of 10 or more. Stewart Way, USA

3985. Tsien, H. S., Influence of flame front on the flow field, *J. appl. Mech.* **18,** 2, 188–194, June 1951.

In addition to providing new information, this paper should stimulate inquiries of greater generality concerning flame shape. The kinematics and thermodynamics of an infinitesimally thick flame front are first examined, yielding a relation between streamline curvature upstream and vorticity downstream of the front. Heat release is defined in terms of pseudostagnation temperature that does not include kinetic energy corresponding to component of velocity tangent to the front.

Scurlock's problem [*Meteor Rep.* no. 19, M.I.T., 1948] of flame spread (for $M \ll 1$) in a constant area, two-dimensional duct is then given a new and simpler treatment, and his problem is also formulated mathematically in an integral equation. Author broadens treatment to include compressibility effects and finds that apparently the fraction f of duct width occupied by the flame has an upper bound if either heat release or upstream Mach number is too large. The calculations show that this limit is reached

when material near the wall has been accelerated, prior to burning, to thermal choking limit $M = 1$. Reviewer feels that this behavior requires further elucidation because (1) in neighborhood of maximum flame spread, assumption made of no pressure drop across flame is no longer tenable and curvature of front or of stream time tends to become infinite; (2) one-dimensionalized treatment may be less accurate for the compressible case; (3) double-valued relation found between f and fraction burned of the stream is not given physical explanation. B. L. Hicks, USA

3986. Rushbrooke, G. S., and Scoins, H. I., On virial coefficients and the Born-Green theory of fluids, *Phil. Mag.* (7), **42,** 329, 582–593, June 1951.

Virial coefficients of the pressure of a fluid, expanded in powers of density, are calculated by the superposition formulas of the Born-Green theory. The second and third are shown to agree with the correct values given by Mayer, but the fourth does not. The discrepancy is traced to the cluster integral involving all interactions. The same type of error arises in linearization of integral equations for the distribution function. Calculations are made for hard spheres.

Virial coefficients of component systems are discussed.

Gilbert W. King, USA

3987. Kling, R., and Leboeuf, R., Velocity of pressure-wave propagation in fuel-injection lines of motors (in French), *Rech. aéro.* no. 19, 59–65, Jan.-Feb. 1951.

A technique is presented for sound-velocity measurements in liquid-filled pipe lines. Experimental values for a series of hydrocarbon fuels covering a wide range of static pressure levels are also given and the effects of fuel-temperature and pipe-line characteristics considered. Andrew Fejer, USA

3988. Watanabe, T., Theoretical determination of the standard performance curves of the internal combustion engine, *Memo. Fac. Sci. Engng. Waseda Univ., Tokyo,* no. 14, 13–15, 1950.

A method is given for predicting the fuel consumption and the brake horsepower of an engine at different speeds and loads. The data at normal output must be known. One set of derived values is given as a comparison with experimental values for a particular engine. A. O. Flinner, USA

Heat and Mass Transfer

(See also Revs. 3755, 3759, 4027)

3989. Brown, A. I., and Marco, S. M., Introduction to heat transfer, 2nd ed., New York, Toronto, London, McGraw-Hill Book Co., Inc., 1951, xvii + 267 pp. $4.50.

Book is intended for third-year undergraduate students, and subject is developed by chapters on steady-state heat conduction, radiation, convection, applications, and a final chapter on variable heat flow.

Steady-state conduction equations are developed from the basic Fourier equation and applied to heat flow through plane composite walls and to flow through composite pipe walls. A method of determining heat flow by mapping curvilinear squares is illustrated, but its basis in the Laplace equation is not indicated. Some worked examples are included.

The chapter on radiation introduces the necessary fundamental concepts and indicates the method of deriving such relations as the Stefan-Boltzman law and shape factors. (Derivations are given in the appendix.) Kirchhoff's law for gray bodies is developed by Hottel's factors for emissivities, and positions of radiating bodies are included. Brief section on solar and gas radiation terminates the chapter.

Three chapters are devoted to the study of heat transfer by convection. The film concept and dimensional analysis are treated in the introductory chapter.

Nusselt's equation for forced convection is derived by dimensional analysis, and several applications are illustrated. The chapter concludes with a section on pressure drops in pipes and Reynolds analogy between heat transfer and fluid friction.

Free convection is treated as a special case of forced convection in which the velocity term in the Reynolds number of the Nusselt equation is modified and the Grashoff number obtained. Short chapters on heat transfer to boiling liquids and condensing vapors round out the basic material on steady-state heat transfer.

The over-all transfer of heat is summed up in a chapter and the more commonly used mean-temperature differences, such as the logarithmic, are presented. The applications are further stressed in a chapter on typical design problems including an application of the method of Tuve for calculating the performance of a dehumidifying coil.

The concluding chapter is headed "Variable heat flow." Fourier's general law of heat conduction is stated but no solutions of the equation are derived. The problem is attacked by the method of dimensional analysis and correlated to the solutions of Fourier's equation for most of the standard unsteady heat-conduction problems.

Reviewer feels that the text should give students a working knowledge of heat-transfer applications and that the approach of the authors to the subject will be well received by most undergraduates. However, he feels that an inclusion in the body of the text of some of the material in the appendixes would strengthen the grasp of the student on basic theory.

William A. Wolfe, Canada

3990. Becker, E. W., Effect of pressure on thermal diffusion in gases, *J. chem. Phys.* **19,** 1, 131–132, Jan. 1951.

As determined by elementary thermal separation, the thermal diffusion factor of gas mixtures indicates a pressure dependence, and increases by a factor of 3, 5, 8, respectively, for the gas mixtures CO_2/H_2, CO_2/CH_4, and CO_2/N_2. The gas mixture N_2/CH_4 shows, on the contrary, a negative dependence. It is shown theoretically that this pressure effect is in large part due to the reality of the gases. Author believes that the special effects observed with CO_2/C_3H_8 by Drickamer, O'Brien, Bresee, and Ockert, are essentially due to the same reason.

C. M. Tchen, USA

3991. Schlichting, H., Some exact solutions of temperature distribution in a laminar flow (in German), *Z. angew. Math. Mech.* **31,** 3, 78–83, Mar. 1951.

Exact solutions are given for the temperature distribution in incompressible flow for the Couette flow, Poiseuille pipe flow, and the flow between two concentric rotating cylinders. Frictional heat is included but the dependence on temperature of the coefficients of viscosity and thermal conductivity is neglected.

G. M. Lilley, England

3992. Shorin, S. N., Radiation heat exchange in an absorbing medium (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 3, 389–406, Mar. 1951.

First section gives basic theory. Second section describes heat transfer in a plane-parallel layer which is moving against an absorbing medium. Two cases considered are given temperature distribution and given heat output. Results are applied in third

section to discussion of heat transfer in boiler furnaces and compared with results of A. M. Hurwitz and V. I. Timofeyev.

D. Ter Haar, Scotland

3993. Eckert, E., The most favorable arrangement of tubes in heat exchangers (in German), *Forsch. Geb. Ing.-Wes. Ausg. B.* **16**, 5, 133–140, 1949/50.

Analysis is based upon data by E. D. Grimison (cross flow) and E. Hofmann (flow parallel to the tube axis). Evaluations are made from two different points of view: (1) On the basis of a dimensionless ratio of the heat transferred per unit time divided by the power required to effectuate the flow through the tube banks (according to A. Weise). This is mainly a criterion for vehicle radiators as it amounts practically to a comparison of pressure drops for the flow outside the tubes in different exchangers with the same frontal area under otherwise identical operating conditions as to the amount of heat transferred, volume rate, temperatures, and properties of the substance. Flow parallel to the tube axis is found to be superior to any cross-flow arrangement. For the latter, differences between tube bundles with tubes in line or staggered are not substantial. (2) Comparison of pressure drops for the flow outside the tubes for a constant heat-exchanger area and tubes of given diameter, for otherwise identical operating conditions as specified under (1). Author finds cross flow superior to the flow parallel to the tube axis, especially in the lower Reynolds number range, as it is emphasized in technical applications (boilers, gas-turbine heat exchangers). Among all investigated cases, the pressure drop decreased as the ratio of pitch [taken normal, (s_1), and parallel, (s_2), to the direction of the flow] to the tube diameter d decreased. Optimal condition was at $s_1/d = s_2/d = 1.25$ for a staggered arrangement. In-line arrangement is slightly less effective at low Reynolds numbers but approaches or surpasses the staggered arrangement in the higher range. Since smaller pitch ratios make the pressure drop generally decrease, only practical requirements introduce a lower limit. All conclusions apply directly to air flow but can be easily extended.

H. H. Korst, USA

3994. Livingood, J. N. B., and Brown, W. B., Analysis of temperature distribution in liquid-cooled turbine blades, *Nat. adv. Comm. Aero. tech. Note* 2321, 73 pp., Apr. 1951.

A set of calculations of heat flow in certain types of turbine blades is presented. The shapes of various sections of blades are approximated, in some cases by using greatly simplified three-dimensional heat flow. For some of these cases the stationary temperature distribution is given by an analytic expression; for others by an infinite series; for still others it is obtained numerically by relaxation methods. In those ways, temperature distributions are determined for various specific values of blade length, temperature of surrounding gas, flow of constant liquid, thermal conductivity of blade, etc. Results of the computations are presented graphically. Although the methods employed are undoubtedly applicable to much more general problems, the presentation is limited strictly to subject indicated by the title.

Franz L. Alt, USA

3995. Schlichting, H., Heat transfer in laminar flow along a flat plate with variable surface temperature (in German), *Forsch. Geb. Ing.-Web. Ausg. B.* **17**, 1, 1–8, 1951.

The Blasius solution of the equations of flow (simplified for the boundary layer of fluids of low viscosity) is inserted in the equation of heat transport. This last equation is then solved for the boundary layer assuming several boundary conditions: e.g., a wall temperature varying linearly or parabolically along the wall. Graphs are given showing the temperature distribution in the boundary layer. It is concluded that the flow of heat cannot be represented by Nusselt's law in the case of temperature gradients along the wall.

H. C. Brinkman, Indonesia

3996. Widell, T., Heat transfer to superheated steam (in Swedish), *Tekn. Tidskr.* **80**, 46, 1165–1166, Dec. 1950.

Experimental data of McAdams, Kennel, and Addoms [*Trans. Amer. Soc. mech. Engrs.* **72**, p. 421, 1950] are compared with theoretical formulas given by Schack ["Der industrielle Wärmeübergang," Düsseldorf, 1940] and ten Bosch [Wärmeübertragung, Berlin, 1936] using new data of Keyes and Sandell on heat conductivity of steam. Schack's formula agrees within 15% with the experimental data up to steam pressures of 150 bar, but ten Bosch gives values too high by as much as 112%. A nomograph is given from which the heat transfer at temperatures between 100 and 600 C can be read for steam pressures between 1 and 150 bar.

D. Ter Haar, Scotland

3997. Ostroumov, G. A., The mathematical theory of steady heat transfer in a circular vertical hole with superposition of forced and free laminar convection (in Russian), *Zh. tekh. Fiz.* **20**, 750–757, 1950.

Assuming superimposed action of free and forced convection, the equations of continuity, motion, and energy may be solved explicitly in the case of laminar flow in a circular hole. The resulting velocity and temperature distributions are given explicitly in terms of Bessel functions. The distribution pattern is shown to be characterized by a parameter essentially equal to the product of Grashof and Prandtl number. Several numerical results are tabulated and shown graphically. Brief reference is made indicating correlation with limited experimental data.

N. A. Hall, USA

3998. Bochirol, L., True specific heat of ferrites of zinc, of nickel and of cobalt, *C. R. Acad. Sci. Paris* **232**, 16, 1474–1477, Apr. 1951.

3999. Schoenborn, E. M., Armstrong, A. A., Jr., and Beatty, K. O., Jr., Thermal properties of certain laminated plastics, *Amer. Soc. Test. Mat. Bull.* no. 174, 54–59, May 1951.

The thermal diffusivity, conductivity, and heat capacity of laminated plastics vary significantly with both the type of filler and type of resin binder. All three thermal properties may be determined rapidly and accurately by unsteady-state methods if the materials may be considered as homogeneous.

From authors' summary

Acoustics

(See also Revs. 3756, 3987)

4000. Trendelenburg, F., Introduction to acoustics, 2nd ed. [Einführung in die Akustik], Berlin, Springer-Verlag, 1950, viii + 378 pp. DM 39.

Text is not so much a revision of the 1939 edition as it is an entirely new book. Most sections have been rewritten and many new ones added to include the wealth of recent developments in acoustics, particularly those reported in the *J. acoust. Soc. Amer.* Book is perhaps best described by comparing it to Morse's "Vibration and sound" and Olson's "Acoustical engineering." It is more concerned with physical illustrations and embodiments than detailed analytical manipulations, in contrast to Morse. On the other hand, it pays greater attention to the physical fundamentals of acoustical instruments than does Olson, but lacks much of his engineering viewpoint. The paucity of detailed

analytical treatment is well offset by the 28 tables and 280 figures, many of which present complicated results in a quite useful form.

While author presents a compendious survey of the whole field, he understandably is best when treating his fields of specialization: Response, distortion, and directionality of transducers; sound spectra and other characteristics of musical instruments and the human voice; visualization of shock waves. The absorption of sound in gases, liquids, and solids is also notably well-handled. Topics such as design and use of electroacoustic devices are only briefly touched upon, but these belong strictly to a text on audioengineering, for which none exists at present. An indication of the material discussed is afforded by listing the chapter (and selected section) headings: I. Vibration and wave theory (simple, coupled, free, forced, self-excited, and nonlinear oscillation; eigenfrequencies of typical vibrators); II. Field quantities and their measurement (pressure, particle velocity, temperature change, intensity, loudness level); III. Generation (radiation theory, analogies based on force voltage, musical instruments, the voice, electroacoustic and thermoacoustic sources); IV. Propagation (velocity, Huyghen's principle, absorption, filters, room acoustics); V. Reception and recording (theory, directionality and calibration of microphones, the ear, phonodisk, phonotape, and phonofilm); VI. Analysis of sound (instrumentation, characteristics of music, speech, and voice); and VII. Appendix (definitions, formulas). Within each chapter are treated such topics as physical and musical acoustics, ultrasonics, instrumentation, and psychoacoustics.

Book is written at senior or graduate level and is not suitable for a primary text. Apart from the language difficulty, no problems are given.

As a reference, however, reviewer can commend it highly. The usual German thoroughness is manifested by the wealth of literature references in the text and in footnotes. In the first fifth of the book there are over 280 citations of the world literature on acoustics, including original sources as well as the latest work. We learn, for example, that Edison conceived the idea of phonodisk reproduction in 1878, the same year that Siemens disclosed the first moving coil, permanent magnet, curvilinear cone loudspeaker. Reviewer has found no misstatements of fact and only two major but obvious typographical errors. The text can be recommended unhesitatingly for the theoretical or engineering acoustician alike. Vincent Salmon, USA

4001. Roberson, R. E., Random noise in an attenuating fluid medium, *J. acoust. Soc. Amer.* **23,** 3, 353–358, May 1951.

Paper treats attenuation to both volume and surface acoustical "white noise." When the attenuation coefficient in a uniform medium is a function of frequency alone, and the noise sources, with random phase, are uniformly distributed in a volume or on a surface, it is easy to express the power spectrum of the sound pressure at any point. Author shows that for fairly general infinite regions of attenuation, the power spectrum is inversely proportional to the attenuation coefficient.

Combinations of a plane-surface noise and volume noise are considered and spectrum is given for various conditions under square law attenuation. Thomas Caywood, USA

4002. Sacerdote, G. G., and Gigli, A., Absorption of sound by resonant panels, *J. acoust. Soc. Amer.* **23,** 3, 349–352, May 1951.

"Resonant panel" means here a nonvibrating perforated panel separated from the wall by an air space. Resonant frequencies calculated as Helmholtz resonator agree well with frequencies of maximum absorption at normal incidence. At random incidence (reverberation), maximum absorption is at a frequency higher by 30–40%; 15–20% is explained by theory.

R. Vermeulen, Holland

4003. Rzhevkin, S. N., Resonance sound absorber with yielding wall, *Nat. adv. Comm. Aero. tech. Memo.* 1273, 21 pp., May 1951.

This detailed analytical investigation of the effect of sympathetic vibration of a resonator front wall on sound absorption is restricted to the case of a single-sheet resonance system with compartments for normal incidence of sound. The general problem is reduced to a concise electrical analog, which clearly illustrates all the special cases previously reported in the literature. Numerical examples are given. The front wall should be fixed sufficiently rigidly to insure that the sound-absorption coefficient is not reduced in a low-frequency absorber of this type.

E. G. Fischer, USA

4004. Sette, D., Measurements of ultrasonic absorption in various mixtures of nitrobenzene, *J. acoust. Soc. Amer.* **23,** 3, 359–363, May 1951.

Following former results, author continues his measurements to obtain new information on acoustical relaxation in liquids. Generally these phenomena, showing an enormous absorption of ultrasonic waves, are caused by irreversible changes of state which occur if swing time of the wave is comparable to the time necessary to establish thermal equilibrium between the external and internal degrees of freedom of molecules. Obviously, transition of energy may take place in liquids in different ways according to the various actions among and inside the molecules. That means, experimentally, the relaxation phenomena are reflected in the shape of absorption in the frequency range or, at a certain frequency, several processes perhaps may be superposed more or less at the same time. In this last case other studies may be done by varying the molecular behavior. Author does it by considering the absorption of binary mixtures in the whole range of mole fraction. He keeps nitrobenzene as one component and changes the other. With regard to the nature of the liquids he comes to interesting conclusions and can complete his former interpretations.

Reviewer notes that absorption depends on temperature and its coefficient may be positive or negative. Consequently, varying the temperature chiefly to lower values, such as offered could give new aspects due to the displacement of minima or maxima.

Peter-Paul Heusinger, Germany

4005. Willard, G. W., Improved methods for measuring ultrasonic velocity, *J. acoust. Soc. Amer.* **23,** 1, 83–93, Jan. 1951.

Some improved sound-wave-interference methods for measuring the longitudinal and transverse ultrasonic velocity in opaque as well as transparent solids may be simply carried out by using the ultrasonic light-diffraction system. The sonic unit of the system is arranged to produce two individual traveling-wave sound beams. Three simple arrangements are described in detail. In case A, one beam travels entirely in a reference liquid while the other beam travels a parallel path in an immersed transparent test specimen. In case B, one beam travels entirely in a reference liquid while the other beam travels an adjacent course through an immersed, transparent, or opaque test prism, and on into the liquid at an angle to the first beam. In case C, the two beams are generated at the equal edge faces of a transparent or opaque isosceles test prism. The two beams traverse the prism to the base, where they are refracted into the test liquid as confluent beams.

In cases A and B, simulated interference (by optical integration), and, in case C, true interference, each give a light and dark

band interference pattern on the screen, whose band spacing is used in calculating the velocity of sound in the test solid.

Advantages of the improved methods are simple preparation of test specimen, directness and simplicity of measurement and calculation, good accuracy, low sonic power requirements.

From author's summary by Martin Greenspan, USA

4006. Temperley, H. N. V., On the velocity of second sound in liquid helium II *Phil. Mag.* (7), **42**, 324, 74–81, Jan. 1951.

Recent measurements by Atkins and Osborne of second sound velocities c_2 confirm the rise below 1 K and indicate an approach to the value $c_1/3^{1/2}$ at $T = 0$ K (c_1 is first sound velocity) in agreement with Landau, and disagreement with Tiza-London type theory. Although the latter theory employs a Bose-Einstein (B.E.) condensation model, the observations do not disqualify such a model, since calculation herein shows only the method of calculation of ρ_s (the density of superfluid), which determines $c_2(T)$, to be at fault. This calculation is carried out with the general relation $E = Ap^n$ between energy E, and momentum vector p, of an elementary excitation and yields $c_2 \propto T^{1/2}$ for small T. A Landau-type calculation, which considers the mean momentum of a body moving with small velocity in statistical equilibrium with the helium to obtain the effective mass of the helium, is carried out with both a perfect B.E. gas and with an assembly of phonons (quantized sound waves) in the helium. In the former case, agreement with a London-Tisza calculation results (although the two methods give different results for an imperfect B.E. gas), but the latter yields $c_2 = c_1/3^{1/2}$. This result is quite independent of Landau's quantum hydrodynamics which therefore receive no support from the observations. However, both methods give the same result applied to a perfect Fermi-Dirac gas, i.e., lack of superfluidity, hence observation of this lack in He^3 does not distinguish the theories. A second Landau calculation of ρ_s based on the mean angular momentum of the small body in equilibrum with the helium is shown to agree approximately with the first calculation for a perfect gas model, but conceivably could disagree for other models. Paul Marcus, USA

4007. Rocard, Y., Guided acoustic waves (in French), *C. R. Acad. Sci. Paris*, **232**, 6, 485–487, Feb. 1951.

By integrating the wave equation with given boundary conditions, author proves theoretically that acoustical waves traveling in an infinite circular cylinder cannot radially propagate into the surrounding medium, if their phase velocity inside of the cylinder is less than that in the exterior medium. Here acoustical pressure sinks quickly to zero with increasing radial distance, remaining, however, at the same phase. Since the phase shift between radial acoustical velocity and pressure is here $\pi/2$, there exists no energy transport in this direction. If acoustical waves travel in exterior medium, they propagate also inside of the cylinder, but with a phase velocity depending on the coupling between both media. Such acoustical wave guides, whose cross section need not be circular, were investigated experimentally by Bell Telephone Laboratories. Sound velocity was artificially reduced below that one in the air by a set of small equidistant disks put on a bar either straight or curved. J. Pachner, Czechoslovakia

4008. Williams, A. O., Jr., The piston source at high frequencies, *J. acoust. Soc. Amer.* **23**, 1, 1–6, Jan. 1951.

King's expression for the velocity potential is transformed exactly for points whose distance from the piston axis is the piston radius. The average of the velocity potential over any circle whose center is on the piston axis and whose radius is the piston radius is also transformed exactly into a simpler expression. Both results are expanded for the case that the piston radius is much smaller than the distance from the piston and simultaneously

$$2\,(\text{piston radius})^6 \ll \text{wave length} \times (\text{distance from piston})^5.$$

Good agreement is found with quoted measurements and some, but not all, quoted calculations. The assumption of a collimated beam near the source is found to be not very accurate.

Eric W. Beth, USA

Ballistics, Detonics (Explosions)

4009. Garnier, M., Remarkable points on a trajectory (in French), *Mémor. Artill. fr.* **24**, 4, 953–1008, 1950.

"Remarkable" are such points where an element (e.g., velocity) has an extremum, a point of inflexion, or a given value; furthermore, the point of contact with the safety curve. Other remarkable points (point of discontinuity = point with an edge of the function for the resistance of the air at the Mach number 1, jumps in the atmospherical values) are dealt with in a former article of the author. Here he makes detailed proposals for calculating the elements for these points, using the first terms of a series in different conditions (curved trajectory, vertical trajectory; step-by-step method, given total trajectory). The treatments are values for the G. H. M. method for calculating trajectories used especially in France. H. Schardin, Germany

4010. Pack, D. C., and Evans, W. M., Penetration by high-velocity jets. I, II, *Proc. phys. Soc. Lond. Sec. B*, **64**, part 4, 376B, 298–310, Apr. 1951.

Part I presents a theory of target penetration in such form that the penetration can be specific by a few parameters which are deduced by simple experiments. Basis for the theory lies in assumptions: Material in jet is solid or liquid of negligible flow stress, or else jet is made up of frangible fragments; and, to first approximation, yield stress of target is negligible compared to jet pressures. Simple relations are worked out on basis of this theory; e.g., using the same jet against different targets gives penetrations varying inversely as square root of target density, etc. Several limitations of theory are discussed and, in particular, a first-order correction is given when target yield stress is not negligible compared with jet pressure.

Part II describes experimental verification of theory. The relations are applied to multiple targets also. On the basis of experiments, theory appears to be adequate, and thus it is possible to predict the penetration of jets into multiple ductile targets. The analysis depends on knowledge of certain parameters, and a procedure to evaluate the necessary parameters is described. The work as a whole appears to be the first successful attempt to formulate a rational, simple theory for the prediction of jet effects on targets. Bruno W. Augenstein, USA

4011. Nas'in, P., Electronics in short-time measurement and high-speed photography and kinematography, *Instrum. Measur. Conf., Stockholm, Trans.* 139–148, 1949.

Paper describes electronic instruments developed and used at the Laboratoire Central de l'Armement in Paris in connection with ballistics studies. Advantages of electronic over electromechanical equipment are emphasized. Block and detailed circuit diagrams, complete descriptions of operation, and applications are given for the following instruments: spiral sweep cathode-ray tube chronograph, decimal counter-chronometer, delay circuits, and spark generators (a) for single front-light exposures, (b) for single shadowgraphic exposures, and (c) for kinematography at rates of 50,000 and 500,000 frames per sec.

Although descriptions of many essentially equivalent instruments and circuits can be found, paper provides an excellent single source from which experimental ballisticians and other investigators of short-duration phenomena can fabricate instruments that perform reliably. J. S. Rinehart, USA

Soil Mechanics, Seepage

(See also Rev. 3833)

4012. Nash, K. L., The elements of soil mechanics in theory and practice, London, Constable and Company, Ltd., 1951, 112 pp. 9s.

This small book is a compilation of four lectures given by the author at Kings College, London. Subjects covered are as follows: Lecture I. History, exploration, classification, and identification of soils; Lecture II. Densities, seepage, consolidation; Lecture III. Shear strength and stability; Lecture IV. Soil stabilization, pavement design, frost action. Each subject is discussed in a general way with frequent reference to British practice. The principal merit of the book is in acquainting the engineer not familiar with soil mechanics with the aims, scope, and methods of the science. Sufficient detail is not given, nor was it intended, to use the book as a text for soil-mechanics design.

Woodland G. Shockley, USA

4013. Hvorslev, M. J., Time lag and soil permeability in ground-water observations, *Wwys. Exp. Sta. Corps Engrs. U. S. Army Bull.* 36, 50 pp., Apr. 1951.

In an effort to lower cost of on-the-site permeability measurements, author proposes that every observation well, pressure cell, or piezometer be converted to, in effect, a variable head permeameter. Paper is lengthy and overly theoretical, and the proposed method is subject to numerous sources of error avoidable with other methods. Author concludes, and this reviewer agrees, "Many of the . . . sources of error are avoided by the commonly used pumping tests . . . but such tests are expensive and time consuming [and reliable—R.G.K.]. Determination of the permeability of soil in situ by means of the time lag of observation wells and piezometers has so many potential advantages that it is hoped that systematic research will be undertaken . . . to develop . . . reliable methods of calibration . . . and methods for . . . correction . . . of the various sources of error in the observations. Until such research is successfully completed, great caution is advisable in the practical application of the results obtained by the method."

Raphael G. Kazmann, USA

4014. Geffen, T. M., Owen, W. W., Parrish, D. R., and Morse, R. A., Experimental investigation of factors affecting laboratory relative permeability measurements, *J. Petr. Technol. Trans.* **3** (192), 4, 99–110, Apr. 1951.

Laboratory studies of relative permeability of sandstone cores were made using three-section plastic-covered core plugs. Tests demonstrated the assembly performed as continuous core under fluid flow. Results are shown for tests with various concentrations of oil, gas, and brine. Study verifies that technique used would permit reliable measurements of flow characteristics. It is indicated that laboratory tests to represent flow in an oil reservoir must be so conducted that the saturation history of the test specimen duplicates that in the prototype.

Woodland G. Shockley, USA

4015. Pogány, A., New method of determining earth pressure, *Civ. Engng. Lond.* **46,** 538, 248–250, Apr. 1951.

A new theory of earth pressure is proposed, in which all the properties of the soil are taken into account by a single parameter. For a retaining wall, this parameter determines the position of a parabola with vortex at the soil surface and passing through the base of the wall. The location and magnitude of the resultant force on the wall are determined from the statical equilibrium of the soil between the wall and the parabola surface, friction being neglected on both the wall and on the parabolic surface.

Author contends that the new theory is not only simpler but in better agreement with experiment of all other theories from those of Coulomb and Rankin to the more recent ones of Terzaghi, Hartwig, and Jaky. Gerald Pickett, USA

4016. Raymondi, C., Experiments with piles, loaded perpendicularly to their axes (in Italian), *Atti Ist. Sci. Costr. Univ. Pisa, Pubb.* 15, 1950, = *G. Gen. civ.* **88,** 7–8, 429–441, July-Aug. 1950.

Data regarding a hundred horizontal loading tests on three vertical concrete piles are given. Results of these tests are compared with results of computations on a theoretical basis, for which three different assumptions for the earth reaction against the piles are made; e.g., reaction constant with depth, reaction increasing linear with depth, and reaction increasing parabolic with depth.

Conclusion is that there is no good correlation between the test results and the theory used, for which several causes are given: the assumption of the earth reaction, the nature of the soil, and the different behavior of soil in repetitive loading.

F. C. de Nie, Holland

4017. Vlasov, I. O., Distribution of velocity of filtration in inclined layers under pressure (in Russian), *Prikl. Mat. Mekh.* **15,** 1, 117–119, Jan.-Feb. 1951.

Author considers the flow of a viscous fluid towards the drill hole up an inclined porous layer of constant thickness bounded above and below by two impervious strata. The flow takes place in a vertical plane, under pressure supplied by the second nonviscous fluid displacing the first from below.

By conformal transformation, expressions are obtained for the components of the velocity anywhere between the dividing boundary of the two liquids and the drill hole in terms of the total discharge of the drill hole. Alexander Hrennikoff, Canada

4018. Shima, S., On the effect of impervious stratum on seepage flow underneath a dam (in Japanese), *J. Soc. civ. Engrs. Japan* **36,** 3, 3 1951.

Two-dimensional seepage problems with the following boundary conditions are treated, taking into consideration the assumptions ordinarily adopted: (1) Vertical impervious layers at both upstream and downstream sides and a horizontal impervious substratum exist; (2) vertical layers above stated and an inclined impervious substratum exist; and (3) an inclined impervious substratum, which issues from a point on upstream surface and extends to infinity, exists. Author could get a chart by which the errors due to the finite length of the box in model tests would be estimated. T. Mogami, Japan

4019. Bjerrum, L., Fundamental considerations on the shear strength of soil, *Géotechnique Lond.* 2, 3, 209–218, June 1951.

The true cohesion and internal friction of a number of saturated clays are determined by a method based on three assumptions: First, that the Coulomb-Hvorslev equation for shear strength holds good; secondly, that for normally consolidated clays there is a unique relation between shear strength and water content; and, thirdly, that water-content change is solely a function of the change in major principal stress. Of these assumptions the first is widely accepted, though not by all workers in the field.

while the second is supported by an increasing body of experimental evidence and its validity; in fact, is closely related to that of the Hvorslev equation. The third assumption is, however, not of general application. It is approximately true for remolded clays of moderate or high plasticity but does not apply to soils with a dilatant structure. This may invalidate some of the author's conclusions, but, for the majority of the clays investigated by him, this third assumption is probably sufficient to allow at least an estimate of the true cohesion to be made. Even with this limitation, the paper is of considerable importance, and particularly in the demonstration that for a clay with a plasticity index of 27 the true cohesion is itself a unique function of water content. A. W. Skempton, England

Micromeritics

(See also Revs. 3870, 3916)

4020. Kuhn, H., Viscosity, sedimentation, and diffusion of long-chain molecules in solution as determined by experiments on large-scale models, *J. Colloid Sci.* **5,** 4, 331–348, Aug. 1950.

The frictional resistance opposing translational or rotational motions of a randomly coiled molecule through a solvent was found by means of experiments on corresponding large-scale models. The geometrically similar models of the random coils whose translational and rotational resistance in a viscous liquid was measured, were made by bending wire in a statistical manner.

Expressions for the sedimentation constant s_0, the diffusion constant D, and the intrinsic viscosity η_{sp}/c have been obtained from the results of these experiments on large-scale models, viz., $s_0 = a_1 + b_1 Z^{1/2}$; $D = (a_2 + b_2 Z^{1/2})Z^{-1}$; $\eta_{sp}/c = Z(a_3 + b_3 Z^{1/2})^{-1}$. Z is the degree of polymerization, and a_1, b_1, a_2, b_2, a_3, and b_3 are constants independent of Z but dependent on the length of the preferential statistical chain element and the average thickness of the polymer chain. These expressions are found to be in agreement with the experimental data. They have been compared with equations by Debye and Bueche and by Kirkwood and Riseman. C. F. Bonilla, USA

4021. Ward, S. G., and Whitmore, R. L., Studies of the viscosity and sedimentation of suspensions, 1, 2, *Brit. J. appl. Phys.* **1,** 11, 12; 286–290, 325–328; Nov., Dec. 1950.

Three sets of experiments are described in which the effective viscosity η of a suspension of solid particles in a liquid with the viscosity η_0 is determined as a function of volume concentration c. The relative viscosity η_r is defined according to Einstein as $\eta_r = \eta/\eta_0 = (1 + kc)$ and was determined by the rising and settling velocities of spheres with 1.4-mm diam and various densities.

The first set of experiments, which used mixtures of methylmethacrylate polymer spheres of sizes between 27 and 208 microns suspended in an aqueous solution of lead nitrate and glycerol, confirmed the predictions of Einstein and Vand for small and medium concentrations, respectively, that the value of the constant k should be independent of the rate of shear, of the absolute size of the suspended spheres, and of the viscosity η_0 of the fluid. A fully unexplained relationship to the degree of uniformity of the suspended spheres was found, however.

The second set of experiments uses the same spheres suspended in oils of various viscosities. The effect of electric charges on the particles caused by the mechanical mixing process at the beginning of the experiment makes the interpretation of the results uncertain. The third set of experiments uses the same method as the first except that the methacrylate particles have odd shapes and sharp edges and corners. The assumption of a wakelike fluid layer, which together with the particle makes an effective solid unit, permits the reduction of the effect of odd-shaped particles to that of spheres. H. A. Einstein, USA

Geophysics, Meteorology, Oceanography

4022. Masuda, Y., Dynamics of baroclinic disturbances in the upper troposphere, *Pap. Meteor. Geophys., meteor. Res. Inst. Tokyo* **1,** 2–4, 188–197, Dec. 1950.

Author develops the equations of motion for small perturbations in a steady flow (in the upper troposphere). Solution of these equations for a barotropic atmosphere (isobaric and isothermal surfaces not coincident), in which certain restrictive conditions are imposed, yields the wave velocity of the perturbation. In addition, the stability of the perturbation is discussed and author shows instability results when decreasing rate of zonal current with longitude exceeds a critical value. Results are compared with those of Rossby and Jaw and found to agree qualitatively. Warren W. Berning, USA

4023. Palmén, E., and Newton, C. W., On the three-dimensional motions in an outbreak of polar air, *J. Meteor.* **8,** 1, 25–39, Feb. 1951.

Vertical and horizontal components of atmospheric motions during the polar outbreak of 3–7, April 1950, over North America are studied. Descending motions are found from considerations of mass continuity and the assumption that the sharp polar front is an impenetrable boundary. Individual trajectories along isentropic surfaces verify the descent of air. The largest rate of subsidence is found at 600 millibars. Strong nongeostrophic winds, located just behind the polar front, are shown to be consistent with dynamic considerations. Vorticity of air columns, similarly oriented with regard to the front, is conserved when the sinking motion is included. Authors conclude that cold-air outbreaks are important modes of exchange of mass, heat, and angular momentum. Lester Machta, USA

4024. Magata, M., On the structure of the jet stream, *Pap. Meteor. Geophys., meteor. Res. Inst. Tokyo,* **1,** 2–4, 175–187, Dec. 1950.

Author selects simple functions which describe zonal wind-velocity distribution in the neighborhood of the tropopause. From these he calculates the pressure and temperature distributions from the geostrophic equation, hydrostatic equation, and equation of state. Reviewer points out that this procedure is the reverse of the more common one by which the zonal wind distribution is determined from the fields of temperature and pressure. Robert G. Fleagle, USA

4025. Rossby, C.-G., On the vertical and horizontal concentration of momentum in air and ocean currents, *Tellus* **3,** 1, 15–27, Feb. 1951.

The analogy between flow in open channels studied in hydraulics and the narrow jets formed by natural causes in the oceans and atmosphere is an intriguing one. Since forces are operative on the rotating earth to maintain these relatively narrow jets of fluid, despite the lateral dissipation of momentum by mixing, it is likely that this absence of restrictive "walls" in the geophysical case may not prevent some of the major features and criteria dominating the open channel from being significant in oceanic and atmospheric jets. It is on this implicit basis that the paper by Rossby is founded. He attempts to explain the observed high concentration of velocities in a narrow vertical layer at or near a surface of density discontinuity in a stably stratified

fluid. In ocean currents such as the Gulf Stream, he observes that very high concentration of velocities occurs at or near the ocean surface, showing rapid decay with depth, while in the atmosphere, the "jet stream" is found just below the tropopause, with strong shear below the jet maximum.

Rossby's analysis is based on the determination of the velocity profile for the situation in which the mass transport in each isopycnic layer is held constant and the momentum transported by the fluid across a normal section is minimal. It is also a basic assumption, in agreement with existing evidence for stable ocean layers, that eddy diffusion of mass properties is one order of magnitude smaller than eddy diffusion of momentum. He shows that under these conditions, whenever the mean velocity falls below a critical value thereby established, the current in a stratified fluid becomes highly concentrated in the vertical. Reviewer believes that a weakness of the work lies in the fact that Rossby does not explain or justify explicitly why the momentum transported by the flow should be minimized. In homogeneous flow in open channels, the criterion of minimal momentum flux and minimal energy flux leads to the same critical velocity regime. In the more complex case of a stratified fluid, however, Craya [Rev. 3923 in this issue] has shown that the critical regime and the resulting velocity profiles are different for minimum energy flux and minimum momentum flux, though the difference is not qualitatively great. To the reviewer's knowledge, sound theoretical or observational evidence has not been put forward, first, to distinguish between the two criteria or, second, to show that either one should be obeyed or approached by the fluid, although the qualitative agreement of Rossby's results with observations indicates the probable validity of such an argument. Since preceding work on atmospheric and oceanic jets has been confined to predicting such flows only in the mean or statistical picture (under the assumption of barotropic flow), if the approach of Rossby can indeed be justified, it represents the first significant step toward predicting the even sharper jets found in day-to-day oceanic and atmospheric studies. Joanne Starr Malkus, USA

4026. Swinbank, W. C., A sensitive vapour pressure recorder, *J. sci. Instrum.* **28,** 3, 86–89, Mar. 1951.

An instrument has been designed to record directly the fluctuations of water-vapor concentration in atmospheric air. Very fine dry- and wet-bulb thermojunctions are used. Fluctuations are measured from suitably chosen, manually preset reference wet-bulb temperature. Instrument is inaccurate for large fluctuations. It was designed to record fine structure of atmospheric water-vapor concentration, but may find other applications, for instance, in humidity control. Serge Gratch, USA

4027. Cowling, T. G., Atmospheric absorption of heat radiation by water vapour, *Phil. Mag.* (7) **41,** 313, 109–123, Feb. 1950.

Collection of experimental data presented on fractional absorption of heat radiation in relatively narrow wave-length ranges, each containing several absorption lines. Correction factors are presented for temperature and pressure. Law derived for variation of absorption with quantity of water vapor indicates a slower extinction of radiation than Elsasser's law. Experimental results are shown to be consistent with Lorenz collision-broadening. Robert Mindak, USA

4028. Gleeson, T. A., A theory of annual temperature variations, *J. Meteor.* **7,** 6, 351–362, Dec. 1950.

Annual temperature cycles of earth and atmosphere are related to the annual cyclic variations of incoming radiation, outgoing radiation, evaporation, vertical turbulent heat transport in the air, and heat conduction in the ground. This is done by setting up and solving appropriate differential equations of heat transfer subject to reasonable boundary conditions at the earth-air interface. Simplifying assumptions are made about the physical processes involved. Theoretical results are checked with observations at four climatological stations; best agreement is found in the ground and in the lower layers of the atmosphere.

Reviewer believes author has explained successfully the major characteristics of the observed annual temperature curve near the surface. Unexplained differences, especially in the upper air, are probably due to neglect of horizontal heat transport and of condensation processes (from author's summary). Extrapolation of results to localities at which temperature records do not exist is impractical because of necessity of knowing other variables (for instance, evaporation, cloudiness, coefficient of eddy conductivity). Franklin I. Badgley, USA

4029. Cowling, T. G., The calculation of radiative temperature changes, *Centen. Proc. roy. meteor. Soc.*, 19–25, 1950.

The assumptions needed in order to derive simple mathematical expressions for radiative flux and radiative heating are briefly discussed. The mathematical expressions are used to draw general conclusions as to the importance of radiative heating in various conditions. The expressions are transformed into forms suitable for numerical or graphical calculations. A method of determining the temperature distribution in radiative equilibrium is outlined.

From author's summary by Donald M. Vestal, Jr., USA

4030. Sherman, L., On permanent perturbations of zonal atmospheric motion, with applications to low latitudes, *J. Meteor.* **8,** 2, 84–94, Apr. 1951.

In the equation of motion, pressure is eliminated by separating the pressure gradient and applying curl operation. The vorticity equation thus obtained is used with the continuity equation and both are simplified by the following assumptions: steady motion, frictionless flow, introducing a coordinate system moving with the perturbation, which is represented by relative quantities so small that the method of small perturbation can be applied. The basic current is assumed zonal with horizontally uniform density. Equations of the problem contain only momentum components and density, and are discussed in spherical coordinates. Density and eastward component can be eliminated and northward and vertical computed as function of the other. This correlation is discussed synoptically for North Pacific high-pressure cell. For a special model of distribution of vertical momentum corresponding to the observed height of the level of nondivergence, the equations are solved and the result represented in diagram of distribution of horizontal momentum components. Z. Sekera, USA

4031. Magata, M., Dynamics of the movement of atmospheric vortices, *Pap. Meteor. Geophys., meteor. Res. Inst. Tokyo*, 38–44, Oct. 1950.

From law of conservation of absolute angular momentum, formulas are derived for possible motion of traveling atmospheric vortexes with vertical axis, embedded in a zonal current. Such vortexes may, e.g., travel with the same speed as the basic current, if there exists no vertical shear in it. For the case of a zonal current with vertical shear some hints are given as to the possibility of treating this problem. Horst Merbt, Sweden

4032. Kinzer, G. D., and Gunn, R., The evaporation, temperature and thermal relaxation-time of freely falling waterdrops, *J. Meteor.* **8,** 2, 71–83, Apr. 1951.

That ventilation enhances evaporation rates is well known, but the exact rate of moisture loss from falling droplets as a func-

tion of drop diameter, environment temperature, humidity, etc., had, previous to this paper, been studied experimentally only by the artifice of supporting droplets in an airstream by wires or fibers. Present paper considers droplets (diameters 0.0016–0.50 cm) falling freely at their terminal velocities, and sets up a theoretical model which is tested by a set of beautifully ingenious experiments. In addition to evaporation rates, equilibrium temperatures of the falling drops and time required to regain equilibrium temperature after a sudden change of environment are also investigated. The droplets are divided into three size ranges: (a) Those that lie in the Stokes-law region and which entrain air, thereby reducing the evaporation rate toward that of a drop at rest (Reynolds numbers 0-7); (b) those falling swiftly enough to ventilate adequately the transition layer of vapor and temperature (Reynolds numbers 7-2000); and (c) those which fall so fast that they are deformed from the normal spherical shape (Reynolds numbers greater than 2000). Since ordinary raindrops belong to the second category, this class constitutes the major subject of the present study.

In working out the theory of evaporation from a freely falling spherical drop, it is necessary to solve the problem of the transport of heat and vapor from the drop when it is exposed to moving air of different velocities. A drop falling through a uniform environment constantly renews the air adjacent to it on its forward face and rejects modified air from its back side. To formulate the heat and vapor transfer, one must consider the steady state resulting from transient transfer to successive packets of fresh environmental air that continually sweep over the drop surface. The fact that the actual turbulent flow around the droplet is not known thus introduces the major uncertainty into the theory, for the time spent in contact with the drop by each environment parcel cannot be explicitly stated. This difficulty is surmounted partially by introducing into the equations of heat and vapor transfer a dimensionless factor representing the increase in transfer due to the sum of the contributions of all the environment parcels passing over the drop. This factor can be calculated theoretically only in the case of completely known flow patterns, in all other cases it must be determined from the results of experiment, although its order of magnitude is known. Evaporation rates, equilibrium temperatures, etc., are then predicted in terms of this factor, which appears in several different relationships, and results are checked by experiment.

The experiments consist mainly of three methods for determining evaporation rates (pertinent to different drop-size ranges), one technique for measuring drop temperature, and one for determining the thermal relaxation time of the droplets when suddenly placed in surroundings of different temperature. The rate-of-evaporation measurements all depend on the known relation between drop mass and terminal velocity, and measure terminal velocity as a function of time. One method records the falling of the drops photographically, the second electrically. The third involves supporting drops in a known-velocity airstream in a calibrated tube. The equilibrium temperature of the drops in an airstream was determined by an ingenious application of the standard schlieren technique of observing small differences in index of refraction. This method when combined with the technique of supporting drops in a known airstream was also used to obtain the thermal relaxation time. The experimental results show encouraging consistency and agreement with the theory.

Joanne Starr Malkus, USA

4033. Brewer, A. W., and Scrase, F. J., Reviews of modern meteorology. I. Meteorological measurements, *Quart. J. roy. meteor. Soc.* **77, 331, 3–32, Jan. 1951.**

A systematic description of the advances and developments in meteorological instrumentation during the past 20 years forms the subject for the first of a series of "Reviews of modern meteorology." Differentiating the instrumental requirements for climatology and synoptic forecasting purposes from those for basic physical research, authors present a comprehensive survey of the latest developments in the measurement of such routine parameters as surface pressure, visibility, surface wind, upper wind, upper air temperature and moisture, and on such research items as radiation, ozone, cloud-drop size, and condensation and freezing nuclei. Special emphasis is given to upper-air meteorology and in particular to observations now being made on the structure and composition of the atmosphere at heights above 20 km. No attempts are made to give details on instrument construction, but the discussion gives thorough coverage to the theory involved and to errors encountered. Authors have also included, in most cases, their suggestions for lines of future research. A valuable list of references is given. Reviewer believes this article fulfills a very definite need in this era of rapid instrumental progress.

J. R. Gerhardt, USA

4034. Barrett, E. W., and Herndon, L. R., Jr., An improved electronic dew-point hygrometer, *J. Meteor.* **8, 1, 40–51, Feb. 1951.**

An electronically operated self-balancing dew-point hygrometer is described. Discussion of basic problems in moisture measurement indicates advantages of dew-point technique. Instrument includes a mirror surface which is maintained at the dew point by a constant cold source combined with an inductive heating system which is controlled by a servo system sensitive to the amount of dew or frost deposited on the mirror surface. Temperature of the mirror is maintained by the servo at a value producing a predetermined thickness of coating by dew or frost as detected by a photoelectric cell, and the system is designed to follow dew-point fluctuations with a minimum of lag. Temperature of the mirror surface is measured by a fine thermocouple just below the surface. Design and operating characteristics are presented, together with a discussion of errors. When several instruments were calibrated in a closed circulation, standard deviation of observations was 0.25 C. Absolute accuracy is not determinable at present for lack of adequate standards. Instrument should have many important meteorological and industrial applications.

Ferguson Hall, USA

Lubrication; Bearings; Wear

4035. Petrusevich, A. I., Basic conclusions from the contact-hydrodynamic theory of lubrication (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk*, no. 2, 209–223, Feb. 1951.

Paper deals with simultaneous solution of both elastic and hydrodynamic problems when two lubricated cylinders are roll-gliding on one another. Deformation of cylinders is calculated numerically from pressure curve by means of the formula of Boussinesq. Due to the thus calculated deformed shape of cylinders, the hydrodynamic equation of pressure is solved numerically for the variation of viscosity of oil by means of formula $\mu = \mu_0 \exp(kp)$. Three cases are calculated for maximum pressure of 10,000 atm and various rolling velocities. Pressure curves take the same course in their mean portions as the Hertz ellipse. On those locations the surfaces of cylinders being in contact with one another, the curves get roundings and touch axis $p = 0$ asymptotically, the quantity of roundings increasing with the velocity. On the outlet side, pressure curves get a second maximum, the value of which increases with the velocity. Behind this maximum, the curve decreases very steeply causing overstresses on the surfaces of cylinders and the precipitate

pitting of lubricated surfaces. Owing to rapid increasing of viscosity when pressure increases, the calculated number of friction in contact reaches large values up to 0.6. However, it is supposed that near the value 0.1 to 0.15, there are new circumstances of gliding, thereby preventing a further increase of this value.

A. von Mohrenstein, Germany

4036. Roach, A. E., Performance of oil-film bearings with abrasive-containing lubricant, *Trans. Amer. Soc. mech. Engrs.* **73,** 5, 677–685, July 1951.

In general, soft materials are desirable for sleeve bearings, since abrasive particles carried by the oil can embed themselves in the surface and thereby produce a minimum of journal scoring and local heating. Unfortunately, any expedient which improves embedability invariably results in reduced fatigue strength. Gridding and the use of overlays tend to combine good fatigue life with good embedability.

Paper describes experiments in which abrasive-containing lubricants were fed to test bearings in an Underwood centrifugal-bearing tester, and the relative embedability was determined from the rate of temperature rise and rate of journal wear. Following results were noted: (1) Rate of bearing-temperature rise and rate of journal wear both increased linearly with concentration of abrasive in the lubricant. (2) When the abrasive particles were larger than the minimum oil-film thickness, rate of temperature rise and rate of journal wear were both proportional to the particle size. (3) For relatively hard bearings with soft overlays, rate of bearing temperature rise and rate of journal wear decreased approximately hyperbolically with increasing overlay thickness. This curve leveled off when the overlay thickness reached about the same dimension as the abrasive particle size. (4) Embedability of gridded bearings (copper or silver layer, bonded to steel back, with babbitt-filled, diamond-shaped indentations) was generally not much better than that of bearings with overlay, and hardly justified the added complexity of gridding. (5) Circumferential oil grooves improved the embedability performance by providing a free channel for abrasive particles to flow to the maximum-clearance region and by improving cooling through better oil circulation. (6) In copper-lead bearings, embedability improved with increasing lead content. (7) In order of decreasing embedability ratings, the materials tested were babbitt, gridded copper, aluminum alloys, copper-lead, copper-lead-tin, and copper.

Eugene M. Simons, USA

4037. Clark, O. H., Woods, W. W., and White, J. R., Lubrication at extreme pressures with mineral oil films, *J. appl. Phys.* **22,** 4, 474–483, Apr. 1951.

The results obtained by the authors add to our understanding of lubrication processes. Between crossed-axis steel cylinders, intact mineral-oil films at pressures up to 215,000 psi were obtained without wear, unless fatty acids or extreme pressure additives were added. Also, with a bearing consisting of a sphere in a spherical shell formed by plastic flow, similar observations could be made.

To account for the fact that hydrodynamic forces here are still effective, the known bulk properties are mentioned by name. The viscosity of mineral oils increases rapidly under increasing pressure; besides, a solidification of the oil film occurs under high pressures.

Hans Drescher, Germany

Marine Engineering Problems

4038. Bonebakker, J. W., The application of statistical methods to the analysis of service performance data, *Trans. N.E. Cst. Instn. Engrs. Shipb.* **67,** part 6, 277–296, Apr. 1951.

Based on a linear relation between torque coefficient and advance coefficient, a relation between speed, power, and revolutions is deduced, the constants of which are determined from a limited amount of service-performance data by means of methods of least square.

H. Lerbs, USA

4039. Lefol, J., Graphical representation of tests of cavitating propellers (in French), *Bull. Assn. tech. marit. aéro* no. 49, 121–134, 1950.

Difficulties encountered in presenting data for cavitation inception (face and back cavitation) and propeller action under cavitating conditions are indicated. Noting that tunnel test results are usually plotted as thrust coefficients and efficiencies vs. advance ratio (V/ND) for different operating cavitation indexes, author suggests plotting inception points on a graph of $\sigma\phi$ (σ is cavitation index based on water speed, ϕ is propeller expanded area ratio) vs. slip or advance ratio. A V-shaped curve results, one leg of the V representing face cavitation inception and the other back cavitation, and the region inside the V indicating no cavitation. A method is presented of reducing tunnel data on propeller action with cavitation to a plot of relative value of the Froude thrust coefficient vs. thrust. Author notes differences between tunnel test data by Gawn reduced to this form and destroyer test data by Dieudonne.

J. M. Robertson, USA

4040 Uniechowski, S. W., Stability curves for new designs, *J. Amer. Soc nav. Engrs.* **63,** 2, 311–318, May 1951.

Author presents a method of computing stability curves for a ship design which has been obtained by "stretching" the dimensions of an existing design. The stretching is such that any point (x, y, z) of the existing form becomes $(\lambda x, \psi y, \mu z)$ where λ, ψ, and μ are the constant ratios of the new to the old principal dimensions. Derivation is straightforward and embodies no approximations. Method allows quick calculation of stability characteristics of new design from curves of existing design which have been obtained by use of integrator or other laborious techniques. Method is not new but apparently is not widely used.

John P. Breslin, USA

Biomechanics

4041. Hill, A. V., Thermodynamics of muscle, *Nature* 167, 4245, 377–380, Mar. 1951.

Paper opened the title discussion at International Congress of Physiology, Copenhagen, August 1950. It summarizes and discusses following facts which theory of muscle operation must explain: Forty per cent of input energy may appear as work. To hold load stationary requires continuing energy flux proportional to speed of muscle. Inert mechanical and thermodynamic properties resemble rubber. If muscle is stretched during active phase, energy disappears, probably into reversible chemical changes. Rate of production of heat is never negative. Author points out that reversible equilibrium thermodynamics cannot explain dynamic equilibria such as those involving continual energy flux for keeping muscle contracted.

Vincent Salmon, USA